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THERMO-ELECTRIC DETERMINATION OF TEMPERATURES BETWEEN 0° AND 200° CENTIGRADE.

By A. de Forest Palmer, Jr.

THE thermo-electric thermometer has been developed, for the most part, in connection with the measurement of very high or very low temperatures and neither the instruments nor the reduction formulæ in common use are available for accurate work in the interval between 0° and 200° C. The apparatus and method, described in some detail below, were, accordingly, developed for the purpose of determining the precision attainable in this interval with suitably chosen thermo-elements. At the outset, the well-known thermo-electric properties of constantin suggested the adoption of the iron constantin element, but, at that time, constantin wire of suitable size could not be obtained, without considerable delay due to the necessity of importation. To save time, wire supplied by the Driver-Harris Wire Co., of Harrison, N. J., under the trade name "Advance," was tried. Analysis of a sample of this wire showed that it contained approximately 55 per cent. copper, 44.4 per cent. nickel and 0.6 per cent. iron. A preliminary calibration of an element, composed of advance wire in conjunction with commercial soft iron wire, by comparison with an uncorrected mercury thermometer showed a nearly linear relation between thermo-electromotive force and temperature when the cold junctions were kept at a constant temperature. The mean electromotive force per degree Centigrade in the range from 0° to 100° C. was found to be approximately fifty micro-volts. These results agree so closely with

the known composition and properties of constantin that it seemed inexpedient to incur the delay of importation and the following observations were, accordingly, made with iron-advance thermo-elements.

OUTLINE OF THE METHOD.

The thermo-elements were connected to a potentiometer by copper wires. The copper-iron and copper-advance junctions were kept constantly at 0° C. in melting ice, while the iron-advance junction was immersed in various constant temperature baths, and the resultant electromotive forces were measured in the usual manner by comparison with a Clark cell. During the preliminary tests of the apparatus considerable uncertainty in its indications was produced by residual thermo-electric effects, *i. e.*, by the electromotive forces generated in the various parts of the potentiometer and at other junctions than those under test. These residual effects were due largely to lack of uniformity in the temperature of the various parts of the potentiometer system; they varied continually, but never abruptly, in magnitude and sign and their resultant magnitude sometimes amounted to ten micro-volts. Since it was impracticable to keep the whole apparatus at a perfectly uniform and constant temperature an auxiliary potentiometer, described below, was introduced in such a manner as to balance the residual effects without, at the same time, impeding the action of the electromotive force due to the element tested. When this device was properly adjusted the uncertainty of a single determination rarely exceeded one micro-volt.

HOT JUNCTION TEMPERATURES.

The iron-advance junction was maintained successively at the transition temperature of sodic sulphate ($Na_2SO_4 + 10H_2O$) and the boiling points of benzene (C_6H_6), distilled water, chloro-benzene (C_6H_5Cl), and aniline (C_6H_7N). The temperature corresponding to each boiling point was calculated from the corrected height of the barometer at the time of observation. Somewhat greater accuracy in fixing these points might have been attained by the judicious use of well constructed and carefully calibrated mercury thermometers, but, as such instruments were not available, reliance was necessarily placed on published determinations of the boiling constants of the various liquids.

Sodic sulphate, marked C.P., was obtained from Eimer and Amend and recrystallized until further continuance of the process produced no change in the transition temperature. Two successive crystallizations usually proved sufficient and, when the heating bath was kept between 33° and 34°C., various samples of the salt exhibited the same transition temperature, within the errors of observation; moreover this temperature remained constant, within the same limits, for long periods of time. As a datum in exact thermometry this point is comparable with the ice point in convenience and accuracy of maintenance and is much superior to any of the boiling points since it is independent of fluctuations in atmospheric pressure. A series of very accurate determinations by Richards and Wells[1] gives 32°.383, hydrogen thermometer scale, as the exact temperature at which transition takes place.

All of the organic liquids, enumerated above, were Kahlbaum products. They were carefully fractionated, in the chemical laboratory of Brown University, until they maintained constant boiling points, and the sample of benzene had stood in contact with sodium wire nearly a year for the purpose of freeing it from water. The boiling points of these liquids have been determined by several observers and the following values, referred to the air thermometer scale, have been adopted as the most trustworthy.

Liquid.	Boiling Point.
Benzene.....	80°.19 + .0455 (H — 760)
Chloro-benzene.....	131°.85 + .049 (H — 760)
Aniline.....	184°.13 + .051 (H — 760)

The first of these values was taken from a paper by Holman and Gleason[2] and the second was calculated from the air thermometer observations of Ramsay and Young,[3] using .00367 for the coefficient of expansion of air. The normal boiling point of aniline is due to Callendar and Griffiths[4] and the pressure coefficient .051 to Ramsay and Young.[5] The temperature of the steam from water boiling

[1] T. W. Richards and R. C. Wells, Proc. Am. Acad., Vol. 38, p. 431, Dec., 1902.

[2] S. W. Holman and W. H. Gleason, Proc. Am. Acad., Vol. 15, p. 237, 1888.

[3] William Ramsay and Sidney Young, Trans. Chem. Soc., Vol. 47, p. 645, 1885.

[4] H. L. Callendar and E. H. Griffiths, Chem. News, Vol. 63, p. 1, January 2, 1891.

[5] William Ramsay and Sidney Young, l. c., p. 655.

under observed atmospheric conditions was taken from Landolt and Börnstein's "Physikalisch-Chemische Tabellen."

APPARATUS.

The construction of the thermo-elements is exhibited in Figs. 1 and 2. *Ad* is a No. 30 B.S. advance wire and *Fe* is a commercial soft iron wire of nearly the same diameter. These wires pass through openings in the cork *K*, Fig. 2; *Fe* inside and *Ad* outside of a glass tube *L* about one millimeter internal diameter, three millimeters external diameter, and twenty centimeters long. They are soldered together at *G* to form the hot junction. The cold junctions *J*, *J'*, Fig. 1, are constructed in essentially the same manner. Both *Fe* and *Ad* pass inside of glass tubes *T*, *T'*, of the same dimensions as *L*, Fig. 2, while the copper wires *Cu*, *Cu*, leadng to the potentiometer, are outside of the tubes. The length of the wires between *C*, Fig. 1, and *K*, Fig. 2, is about one meter. Both of these wires and also the copper wires *Cu*, *Cu* are insulated by passing through small rubber tubes and all of the wires and tubes are held firmly in place in the corks *C* and *K* by sealing wax caps *W*, *Y* so that the element can be easily removed from one heating bath to another. Three elements, constructed from different portions of the same samples of wire, were tested. The iron-advance junctions were entirely independent of one another but all of the copper-iron and copper-advance junctions were brought together in the tube *U* of the cooling apparatus, Fig. 1, for the purpose of keeping them at a uniform temperature. For the sake of clearness only one pair of junctions *J*, *J'* is delineated in Fig. 1, but in practise the three pairs were held firmly together by the cork *C* and a rubber band around the lower ends of the tubes *T*.

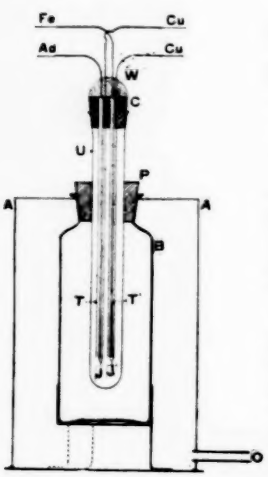

Fig. 1.

The cooling chamber, Fig. 1, consists of a wide-neck glass bottle *B*, about 600 c.c. capacity, centrally supported in a tin plate tank

AA, 22 cm. high and 15 cm. diameter. A strong test tube *U*, filled to about one third its capacity with paraffin oil, is supported, with about two thirds its length in the bottle *B*, by the cork *P* and serves as a bath for the cold junctions *J*, *J'*. The tank *AA* was placed inside of a large wooden box covered with asbestus to shield it from direct radiation from the heating apparatus at the hot junction. Clear pond ice, free from specks and air bubbles, was crushed to about the size of large chestnuts and carefully washed with distilled water, while contained in a glass funnel to allow free drainage. About two thirds of the anular space between *U* and *B* was filled with ice thus prepared and the remaining space was filled with distilled water. The space between *A* and *B* was filled with ordinary crushed ice. After standing twenty or thirty minutes, with occasional agitation of the ice and water in *B* by means of the tube *U*, which was slightly raised from its normal position for the purpose, the temperature of the oil in *U* became uniform and remained perfectly constant, within the limit of sensitiveness of the thermo-elements, as long as the apparatus was in use; ten or twelve hours on some occasions. The drainage tube *O* was normally closed by a rubber tube and pinch cock but it was occasionally opened to drain off water, due to melting of the ice in *A*, after which the space between *A* and *B* was again filled up with crushed ice. When the iron-advance junction was immersed in a similarly constructed bath, charged with various samples of pond ice prepared as described above, the resultant electromotive force of the system was invariably too small to be detected with the galvanometer used in these experiments, although it easily showed a difference of potential of one tenth of a micro-volt between its terminals. It was, therefore, concluded that the temperature of the bath *B* represented the true melting point of ice within the limit attainable by the present method.

The sodic sulphate bath is illustrated in Fig. 2. *DD* is a glass beaker, 18 cm. high and 10 cm. diameter, nearly filled with water and provided with a german silver heating coil *HH*. The test tube *E*, 20 cm. long and 3.5 cm. diameter, is supported axially in the beaker by a clamp stand not shown in the figure. *F* and *S* are smaller test tubes supported concentrically with *E* by the corks *N*

and *M*. *S* contains paraffin oil, that serves as a bath for the iron-advance junction, and *F* contains sodic sulphate, prepared as described above. Sufficient salt was introduced to keep *F* filled nearly as high as the surface of the water in *DD*. The anular air space between *E* and *F* serves to render the interchange of heat between the two tubes more uniform and aids materially in maintaining a constant temperature. A mercury thermometer was immersed in the water in the beaker and the temperature quickly raised to about 33° C. with a Bunsen burner. The coil *HH* was then connected to a storage battery through an ammeter and rheostat and the current strength was adjusted so that the temperature of the bath remained nearly constant. The tube *F* was cautiously heated, over a Bunsen flame, until the sodic sulphate began to melt; it was then placed in the water bath, as indicated in Fig. 2, and the salt thoroughly stirred, with the tube *S*, until equilibrium was established and the temperature remained constant. The essential features of this apparatus and method of manipulation are the same as those adopted by Richards and Wells[6] in their determination of the transition temperature.

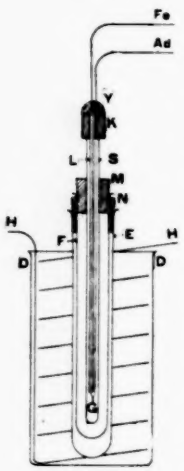

Fig. 2.

Vapor jacketed hypsometers were employed with benzene and water. These instruments were of the well known type, adapted to the determination of the boiling points of accurate mercury thermometers in the vertical position, and were provided with tubular oil baths, similar to *S*, Fig. 2, in the place usually occupied by the thermometer. The instrument used with water was made of tin plate and held about one liter of water. It was not provided with a condenser and hence volatile impurities in the water would produce a gradual change in the boiling point. Since the observations give no evidence of such a change, it was assumed that the distilled water employed was free from such impurities. A sheet copper hypsometer, provided with an inverted condenser, was used with benzene and the condenser was occasionally turned over to test the

[6] T. W. Richards and R. C. Wells, l. c.

constancy of the boiling point. Both of these instruments were jacketed on the outside with felt packing and asbestus and contained a quantity of sharp-pointed bits of copper wire to insure quiet boiling and prevent super-heating.

Chloro-benzene and aniline were boiled in the apparatus illustrated in Fig. 3, some features of which are due to Barus.[7] *T* is a glass boiling tube, 36 cm. long and 4.5 cm. internal diameter, surrounded by a plaster of paris muffle *MM*, 20 cm. long and 15 cm. diameter. The muffle was protected by the tin plate can in which it was cast and, together with the other parts of the apparatus, was supported by a clamp stand not shown in the figure. The top of the boiling tube was closed by a cork *P* that supported a tubular oil bath *S* and an inverted condenser *C*. The muffle and the vapor in the upper part of *T* were protected from the direct radiation from the ring burner *B* by an asbestus diaphragm *A* and the lower end of *T* was surrounded by fine brass gauze *G* to insure uniform heating. A number of short capillary tubes, closed at the upper end and supported vertically in the liquid, caused the ebullition to progress regularly and quietly. The surface of the liquid was never allowed to fall below the diaphragm *A* and the burner was so adjusted that the condensation line in the condenser *C* remained at an approximately constant height, 10 to 15 cm., above the cork *P*. Under these conditions the temperature of the vapor remained perfectly constant so far as could be detected by the thermo-elements. During the course of the experiment the constancy of the boiling point was tested by inverting the condenser and allowing the liquid to distill over. The purity of the liquids is attested by the fact that this process never produced a definitely observable change in the temperature of the vapor, although more than half the volume of liquid was allowed to pass over in some cases.

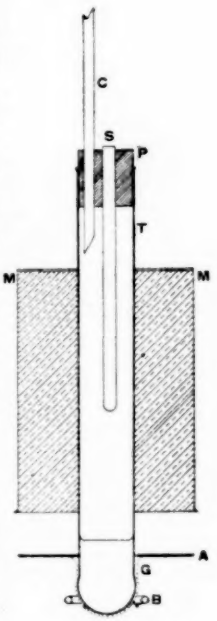

Fig. 3.

[7] Carl Barus, Bulletin No. 54, United States Geological Survey, 1889.

Fig. 4 is a schematic diagram of the potentiometer system and connections. The resistances R_1, R, R', and the slide wires A–A' and E–E' constitute the primary potentiometer and are connected in series with a single storage cell B through the double pole switch S. The variable high resistance R_2 and the slide wire F–F', connected in series with a single dry cell B' through the reversing key D, constitute the auxiliary potentiometer, used to balance the residual thermo-electric effects. The thermo-element T is connected to the primary potentiometer, through the three way double pole switch W, the double pole double throw switch P, the galvanometer G, and the length F–M of the auxiliary potentiometer wire, as indicated by the full lines in the diagram and its electromotive force is balanced against the fall of potential along HA and the resistance

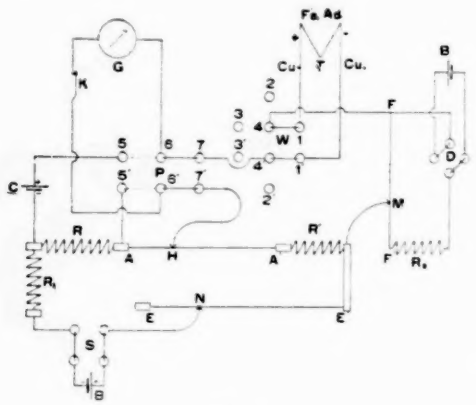

Fig. 4

R'. For the sake of clearness only one of the elements is represented in the drawing. The other two are connected to the points 2, 2′ and 3, 3′ respectively and the interchange of elements is easily effected by moving the connection bars 4–1 and 4′–1′ of the switch W. When the switch P is thrown to the position indicated by the dotted lines 5–6 and 5′–6′ the Clark cell C is connected to the primary potentiometer, through the galvanometer, and its electromotive force is balanced against the fall of potential through a fixed resistance R of about 2,000 ohms. The slide wire A–A' was carefully calibrated and the resistances R and R' were accurately

determined in terms of its mean resistance per unit length. The contact H was moved by an endless wire belt actuated by a milled head, situated at the end of the slide wire, and its position was determined by a vernier reading directly to one tenth of a millimeter.

Observations.

With the switch P, Fig. 4, thrown to the position 5–6, 5′–6′ and the switch S closed, the resistance R_1 and the sliding contact N were so adjusted that the galvanometer showed no deflection when the key K was closed. P was then thrown to 6–7, 6′–7′, R' and R_1 changed reciprocally, and H moved along the wire until balance was again produced. The connection bar 4–1 was then moved to the position indicated by the dotted line 4–4′, S was opened, and the residual electromotive forces were balanced by properly adjusting the commutator D, the resistance R_2, and the sliding contact M. These operations were repeated, in the above order, until exact balance was observed in each of the three cases without changing the potentiometer adjustments. Usually two repetitions sufficed for this purpose and, as the various switches could be worked rapidly, the necessary operations caused very little loss of time and inconvenience.

When these adjustments have been made it is obvious that the resultant electromotive force (V), in micro-volts, of the element connected to the potentiometer is given by the expression

$$V = (R' + x)\frac{E}{R} \times 10^6 \qquad (1)$$

where E is the electromotive force of the Clark cell at the time of the observation, R and R' are resistances, designated in Fig. 4, expressed in terms of the resistance of one centimeter of the slide wire, and x is the corrected length of the wire between A' and H. The electromotive force of the Clark cell used in these experiments was given by the formula

$$E = 1.438 - .001(t - 17) \qquad (2)$$

where t is the temperature of the cell, and the resistance R was equivalent to 148,870 cm. of the slide wire. Substituting these values in (1) gives

$$V = (R' + x)\,[9.659 - .007\,(t - 17)] \qquad (3)$$

Hence, a change of one millimeter in x is equivalent to a change of a little less than one micro-volt in V and as the contact H can be easily set and read to one tenth of a millimeter the limit of sensitiveness of the apparatus is about one tenth of a micro-volt.

During all of the observations the copper-iron and copper-advance junctions were maintained at 0° C. in the bath illustrated in Fig. 1 and, before each observation, the mixture of ice and water in the bottle B was thoroughly stirred. The first series of measurements was made with the iron-advance junction immersed in the sodic sulphate bath, thus producing 32°.383 difference in temperature between the hot and cold junctions. Before each observation the salt in the tube F, Fig. 2, was thoroughly stirred and immediately after balancing the potentiometer the temperatures of the Clark cell and the water bath D were noted. The three elements were tested in succession; about five independent observations being taken with each element while the other two were at room temperature. Two series of observations were made in this manner before the elements were raised to a higher temperature and two similar series were taken after all of the observations at other temperatures had been completed. The average deviation of a single observation from the mean electromotive force of the corresponding element was 0.5 of a microvolt; an amount that corresponds to a change of about one hundredth of a degree in the temperature of the element. Nearly three months elapsed between the first two series of observations and the last two. During this interval the elements were subjected to frequent abrupt changes in temperature sometimes amounting to nearly 200° C. The mean electromotive force derived from the first two series differs from that derived from the last two by only three tenths of a micro-volt. Hence, the temperature baths can be reproduced within somewhat less than one one hundredth of a degree and the thermo-electric properties of the elements are permanent within the errors of observation.

Essentially the same procedure was adopted when testing the elements in the other temperature baths. Care was taken to keep the liquids boiling at a uniform rate, as indicated by the height of the condensation level in the condenser, and after each observation the height and temperature of the barometer were accurately ob-

served. Owing to fluctuations in the atmospheric pressure, and consequently in the temperature of the baths, the individual observations corresponding to the several boiling points do not agree so closely among themselves as do the observations at the transition temperature of sodic sulphate but in all cases the electromotive force of the element followed the changes in temperature of the baths with satisfactory regularity and no detectable change in boiling point due to distillation was observed.

The observations were reduced with the aid of a table, showing the corresponding values of t and log $[9{,}659 - .007\ (t - 17)]$, and the mean results, corresponding to the various temperature intervals, are brought together in Table I., where the data relative to each element are entered in a separate section. The first column gives the number of individual observations from which the mean electromotive force of the element (V_T) was calculated and the second column gives the values of V_T, in micro-volts, thus obtained. The third and fourth columns contain the mean corrected height of the barometer (H) and the mean temperature (T) of the iron-advance junction respectively.

TABLE I.

N	V_T	H	T	θ	T'	ϵ	ϵ'
Element No. 1. $V_{100} = 5{,}188.9$							
20	1,629.5		32.383	31.405	32.385	.0067	.021
23	4,131.4	761.76	80.270	79.618	80.265	.0091	.011
16	5,192.0	761.50	100.055	100.060	100.058	.0098	.010
28	6,916.8	760.20	131.86	133.30	131.81	.0478	.036
18	9,776.6	755.39	183.89	188.41	183.87	.0211	.011
Element No. 2. $V_{100} = 5{,}190.2$							
22	1,629.9		32.383	31.403	32.383	.0115	.036
11	4,133.1	761.78	80.271	79.632	80.279	.0073	.009
16	5,193.2	761.33	100.048	100.058	100.056	.0217	.022
15	6,915.4	758.34	131.77	133.24	131.75	.0286	.022
13	9,785.6	756.85	183.97	188.54	183.99	.0191	.010
Element No. 3. $V_{100} = 5{,}194.1$							
20	1,631.1		32.383	31.402	32.382	.0120	.037
11	4,135.2	761.73	80.269	79.614	80.261	.0089	.011
15	5,196.0	760.96	100.035	100.038	100.037	.0099	.010
15	6,919.2	758.08	131.76	133.21	131.72	.0414	.032
13	9,792.3	757.35	183.99	188.53	183.98	.0119	.006

EMPIRICAL FORMULÆ.

For the purpose of developing an empirical formula to express the relation between the observed electromotive force of the elements and the temperature of the junctions let

$$\theta = 100 \frac{V_T}{V_{100}} \tag{4}$$

where V_T and V_{100} represent the electromotive forces generated when the iron-advance junction is maintained at the air thermometer temperatures $T°$ and 100° respectively and the copper-iron and copper-advance junctions are at the temperature of melting ice. The values of V_{100} given in Table I. were obtained by interpolation between the results of two series of observations at the boiling point of water, during one of which the atmospheric pressure was above normal and during the other below normal. θ may be called the thermo-electric temperature of the hot junction and since V_T is identically zero when T equals zero it is obvious that θ would be equal to T at all temperatures if V_T and T were connected by a linear relation. A large scale plot having V_T and T for coördinates shows scarcely perceptible deviation from a straight line but the calculated values of θ entered in the fifth column of Table I. show differences between T and θ varying from zero to about two per cent.

Since the difference $T - \theta$ changes sign as T passes through 0° and 100°, it was thought that the curve representing the relation between T and V_T might be a parabola, such as is usually found in thermo-electric measurements, but a reduction of the observations on this assumption led to larger discrepancies than those obtained with the linear equation. From an inspection of these results it was apparent that the true T, V_T curve was situated between a straight line passing through the points $T = 0$ and $T = 100$ and a parobola passing through the same two points. An equation of the third degree in the form

$$T - \theta = \theta(\theta - 100)(P\theta - Q) \tag{5}$$

where P and Q are constants, was consequently tried and it was found to represent the observations with sufficient precision. This equation cannot be considered as a physical relation since it gives

three values of θ, and consequently of V_T, corresponding to each value of T, whereas no such phenomena have been observed. It would, perhaps, be more rational to express $T - \theta$ as a function of T rather than of θ but, when tried, such a formula did not reproduce the observations so accurately as equation (5) and it greatly increased the labor of computation. Equation (5) has consequently been adopted as a purely empirical formula for the purpose of reducing the thermo-electric temperature θ to the air thermometer temperature T.

The constants P and Q were calculated from the mean values of θ and T corresponding to the transition temperature of sodic sulphate and the boiling point of aniline. The results thus obtained are given in Table II.

TABLE II.

Element.	$P \times 10^9$	$Q \times 10^7$
No. 1	1,163	4,905
No. 2	1,152	4,911
No. 3	1,165	4,920
Mean	1,160	4,912

The mean values of these constants were adopted and the accuracy of the reduction formula thus obtained was tested by reference to the actual observations. For this purpose equation (5) was put in the form

$$T = \theta + D \tag{6}$$

where

$$D = \theta(\theta - 100)\,(116\,\theta \times 10^{-8} - 4912 \times 10^{-7}) \tag{7}$$

and the curve represented by equation (7) was plotted on such a scale that values of D could be read off directly to 0°.01. To give an idea of the magnitude of the correction term D and the manner in which it depends on θ a few of the calculated values from which the above curve was plotted are given in Table III.

TABLE III.

θ	D	θ	D
0	0	100	0
25	0.867	125	−1.082
45	1.086	150	−2.379
75	0.758	175	−3.783

The individual observations were all reduced by equation (6) with the aid of the curve determined by (7) and the mean results thus obtained for each element, at each of the temperatures employed at the iron-advance junction, are entered under T' in Table I. In each case the actual probable error of a single observation and the percentage probable error of a single observation, entered under e and e per cent. respectively in Table I., were calculated in the usual manner from the residuals obtained by subtracting each of the values of T obtained by reduction from the corresponding value determined by the known temperature of the bath surrounding the hot junction.

CONCLUSION.

The results given in Table I. show conclusively that temperatures, in the interval from 0° to 200° C., can be determined, within one twenty-fifth of one per cent., with the iron-anvance thermo-element when the residual thermo-electric effects are suitably eliminated. Most of the necessary apparatus is to be found in every well-equipped physical laboratory and the remainder can be very easily constructed by any intelligent person. Although proper calibration requires determinations of the electromotive force corresponding to three independent temperature intervals, the necessary observations and reductions are not unduly tedious. The fixed temperatures adopted in this investigation, namely — the melting point of ice, the transition temperature of sodic sulphate, the boiling point of water and the boiling point of aniline — can be easily reproduced and maintained constant with sufficient precision. The materials required for this purpose are cheap and can be easily purified; aniline being the only substance likely to give trouble in this respect. The purity of the aniline can, however, be very easily tested by making observations during distillation, when slight impurities will produce a marked change in the boiling point.

When determining temperatures, after the element has been calibrated, the reductions can be facilitated by combining equations (1) and (4) and calculating θ directly to the formula

$$\theta = (R' + x)\frac{E}{RV_{100}} \times 10^8 \qquad (8)$$

with the aid of a table or curve showing the variation of the factor $E \times 10^8/RV_{100}$ with the temperature of the Clark cell. If a Weston cadmium element is used in place of the Clark cell the correction due to changes in its temperature will generally be negligible.

Under favorable conditions, temperatures, in the interval considered, can be somewhat more accurately determined with a properly constructed and carefully calibrated mercury thermometer than by the method herein described. But such an instrument requires very careful handling and gives reliable results only when suitable corrections are applied to its indications. In many cases it is very difficult if not impossible to determine these corrections with sufficient accuracy to obtain the highest precision in the measurement of temperature. In such cases the thermo-element is quite as accurate and often much more convenient than the mercury thermometer.

BROWN UNIVERSITY,
March, 1905.

THE VOWEL A^o (AS IN RAW), O (AS IN RODE), U (AS IN RUDE).

BY LOUIS BEVIER, JR.

MY researches into the nature of the vowels, conducted with the aid of the phonograph, and a recording instrument, have been published from time to time in the PHYSICAL REVIEW, giving results for *a*, a^r, *e*, i^r, and *i*. Since the publication of the last article, "The Vowel *i* (as in pique)," September, 1904, I have been taking records as opportunity allowed of the series a^o, *o*, *u*. The results reached are given below. A few preliminary words of explanation are needed.

The vowel a^o (as in raw) is a sound which in general is uniform in its pronunciation. As ordinarily pronounced, it is a single sound with no important shift of vocal position during its utterance. The results reached therefore by the analysis of a^o-curves are quite consistent, and accord precisely with what was to be anticipated. In general a^o is different from *a* (as in father) in a somewhat lower pitch both of the main resonance (1150:1100) and of the secondary resonance (700:650).[1]

Hence it is, that if *a* is recorded in the phonograph and the speed in reproduction is reduced the sound issuing is a clear and normal a^o. If the speed is quickened an a^o becomes *a* in reproduction, and if still further quickened the result is a species of a^r, but not a normal one. The reason for this is apparent by comparing the acoustic analysis of *a* and a^r.[2]

The vowel *o* is by no means so easy. In the first place, as spoken in America, it is usually a diphthong, beginning with an *o* and ending with an *u* glide. I have attempted to record and analyze the *o* sound simply, but in spite of great care the results with different voices do not show a close correspondence, a fact which greatly

[1] PHYSICAL REVIEW, April, 1900.

[2] Cf. PHYSICAL REVIEW, March, 1902.

increases the difficulty of definite description, but which was to have been expected, since recent phonetic studies reveal the same differences and varieties of sound grouped together as *o*'s.

With *u* another difficulty appears. Here there is often added (particularly in feminine utterance) a very high resonance, probably due to the closeness of the lips, which does not properly characterize the *u* sound. And this resonance is no more stationary for *u* than is the characteristic high resonance of *i* for that vowel,[1] but may vary within wide limits or be entirely wanting, without depriving the sound of a satisfactory *u* quality.

The most casual observation by ear, by instrumental registration of lip position, or by acoustic analysis, reveals the fact that *u* may be uttered with varying degrees of closeness and still be heard as *u*. Individuals even pronounce *u* in certain words, with an opening which brings it very near to a close *o*. It is therefore idle to expect that the curves will show as constant a character as the curves of sounds which are themselves more constant.

A comparison of the tables of analyses of *o* and *u* with those of a^e and *e*, will show how much greater the agreement in detail is in the case of the "bright" vowels, than in the "dark" vowels which we are now studying.

Passing these generalities I present herewith tables of analyses for a^o, *o*, and *u*, as computed from phonograph curves. For explanation of the methods used, I refer the reader to the preceding articles of this series, particularly to the article of Professor Prentiss.[2]

In the table of a^o analyses, it will be observed that generally there is strong resonance at or near a frequency of 1,000 vibrations to the second. This is the chief and most characteristic region of resonance for the a^o timbre. On a fundamental of 128 vibrations to the second, VIII. (1,024) will be strong; at 256 it will be IV., at 320 III., etc.

Again there is a marked secondary region of resonance culminating at about 640 to the second. Taking the same chord-tones, we shall find that on a fundamental of 128 to the second V. will be

[1] Cf. PHYSICAL REVIEW, November, 1902.

[2] PHYSICAL REVIEW, November, 1902.

strongly reinforced, and on a fundamental of 320 II. Comparing this result with the analyses of *a*, a^o is seen to be a low-pitched *a*.

One further characteristic must be noted. In the series *a*-a^e-ε-i^e-*i*, there is seen to be a generally diminishing amplitude in the curves. Thus, save where the fundamental or chord-tone is strongly reinforced, the traces for a^e has less amplitude thus *a*, and so on till for *i* the trace does not generally travel far on either side of a median straight line. A^o on the other hand gives a trace of great amplitude, the greatest of any of the vowels studied except *u*. This is plain for the series *a*, a^e, ε, i^e, *i* by inspection of the sample curves shown.

The reason *u* shows so large an amplitude is not because its resonance is loudest, but because it is concentrated chiefly in the lowest frequencies, in the chord-tone or first upper partial, where great amplitude does not involve great energy. In the experi-

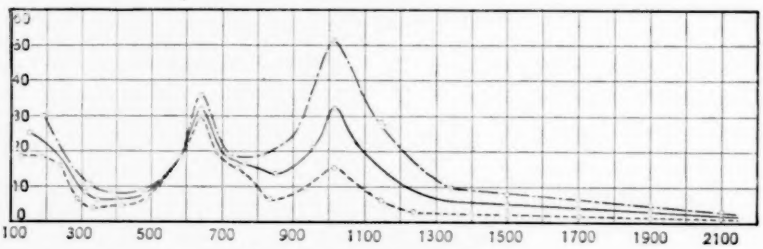

Fig. 1. A^o Resonance Curve.

mental work no effort was made to have the vowels sung with uniform loudness, hence the above comparison is merely a rough and ready test of average.

The analyses of *u*, which for convenience I discuss next, show some interesting facts. This vowel is to be described acoustically with negative rather than with positive characteristics. The mouth cavity in the normal *u*-position seems to have no marked resonance higher than about 210 vibrations to the second. In low tones of men's voices, this may be heard as a strong reinforcement of II. *cf.* the first two analyses given in the tables, voices one and four at 113. Obviously however as the chord-tone rises, this resonance will shift from II. to I.

Where the chord-tone lies above, say, 250 vibrations to the

second, the table shows great inconsistencies. This is due in part to inconsistencies of utterance, and in part to the fact that where no

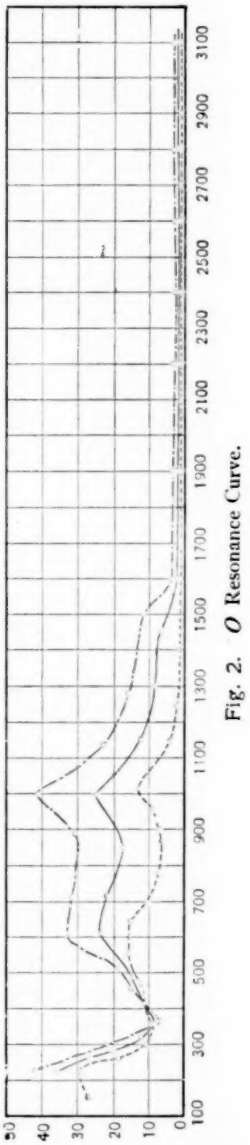

Fig. 2. *O* Resonance Curve.

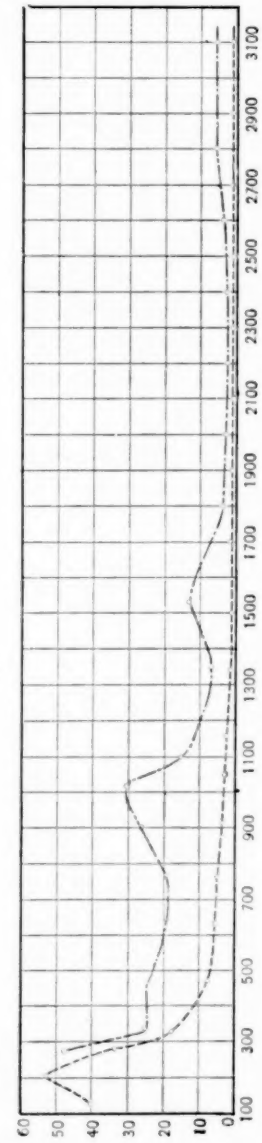

Fig. 3. *U* Resonance Curve.

resonance frequency is present in decided amplitude, the error due to the favored rates of the phonograph diaphragm is greatly increased.

Though I give the resonance curve therefore as before, I regard only the line for men as of great significance. One fact, however, is shown by the upper line, *i. e.*, the presence in many feminine *u*'s of a high resonance in the neighborhood of 3,000. This is present in some cases in considerable amplitude, so plainly in fact that many of the tones reveal it to the unaided eye at a casual examination. To this I have already alluded. I have not observed this in any of the men's *u*'s. Of course with low fundamental chord-tones, such a high resonance would not be prominent in the trace. Still, were it a characteristic mark of *u*-quality, its presence should be detected and its amplitudes should be capable of measurement. That it is an accidental resonance, whose absence does not impair the true *u*-quality is further shown by the fact, that many women's *u*'s are without it, and that its position is free to shift within considerable limits of pitch. In the resonance-curve computed on averages it raises slightly the end of the line all the way from 2,800 to about 3,300 as is seen in Fig. 3, page 83.

The normal *u* then will have strength in the chord-tone and in the first upper partial on lower fundamentals, and chiefly in the chord-tone on higher ones. The ear identifies it by the absence of marked upper resonances.

The analyses of the *o* curves are not so consistent, and their interpretation presents no little difficulty. If the individual analyses are studied one by one, the range will be found to be very wide, running from a typical and normal a^o to a fully closed *u*. For example, voice 6 at 144 is a good a^o-record, while voice 1 at 144 gives a record which is a normal man's *u*. Intermediate between these extremes all shades seem to be represented.

One reason of this is, I conclude, that there was little uniformity of utterance among the voices tested. The diphthongal character of *o* allows anything to pass with the ear, from $a^o\underset{\sim}{u}$ to a true *o*. If the waves analyzed were taken from the earlier part of the utterance, the analyses should be more or less like a true a^o, if from the latter part, more or less like a true *u*. I conclude from this that the intermediate examples represent approximately the truth, and a true monophthonal *o* may be described phonetically as follows: It possesses a strong resonance at about 600 to the second, but none

higher, also a resonance heard either as a chord-tone or as a first upper partial at about 225. These two resonances seem to be characteristic. A good example of such an *o* will be seen in voice 4 at 128, or in voice 5 at 288. In the first II. and V. are strongly reinforced, in the second I. and II. These facts are shown in the *o*-resonance curve (Fig. 2, page 83), but since this is computed from averaging all the records analyzed, it shows also a resonance at 1,000, due to the presence of many records which shade toward a^o.

If these conclusions are correct, it becomes clear why reproducing a phonographic *u* at an increased speed, or an a^o at a reduced speed, will not yield a satisfactory *o*-sound. In view of the inconsistencies of the records analyzed it is desirable that further researches be made with *o*-records, great care being taken in recording normal *o*-sounds, and also in selecting waves for measurement from different parts of the trace for careful comparison. This is of course absolutely essential in the study of diphthongs.

THE VOWEL a^o.

		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.	XVII.
		113	226	339	452	565	678	791	904	1017	1130	1243	1356	1469	1582	1695	1808	1921
1	119.8	32.0	16.8	1.1	3.7	16.6	15.8	3.6	1.2	2.9	1.3	0.4	0.7	0.8	1.4	1.2	0.3	0.2
4	69.9	2.0	30.7	2.1	2.0	5.9	18.7	3.9	7.3	13.4	1.1	1.6	1.5	2.7	0.7	0.7	1.2	0.2
		128	256	384	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048	2176
4	156.5	6.1	11.7	2.3	2.8	10.4	20.9	3.7	16.2	11.9	1.8	0.9	2.7	0.7	2.0	1.3	1.2	0.7
6	47.7	8.2	23.5	0.6	1.9	30.2	2.7	5.7	10.9	4.4	1.9	0.2	2.1	1.9	1.5	1.9	1.0	1.4
		144	288	432	576	720	864	1008	1152	1296	1440	1584	1728	1872	2016	2160	2304	2448
1	133.3	1.8	12.1	6.2	52.7	2.1	7.1	7.2	2.1	1.8	1.1	2.0	0.6	0.6	0.5	1.2	0.4	0.5
4	253.2	21.5	5.4	2.1	8.1	20.4	3.0	15.2	10.4	2.9	2.5	1.3	0.8	0.8	0.4	0.4	0.2	0.4
6	139.5	43.1	4.4	1.8	9.4	4.9	4.3	9.5	1.4	1.1	1.6	0.3	0.5	0.9	0.9	0.9	1.4	2.2
		160	320	480	640	800	960	1120	1280	1440	1600	1760	1920	2080	2240	2400	2560	2720
1	243.3	12.5	7.6	3.8	52.6	5.1	2.3	1.4	1.2	0.9	1.0	1.1	1.2	0.6	0.9	0.9	0.5	1.9
4	265.6	21.2	2.2	2.4	19.4	10.6	9.3	22.2	2.2	2.8	1.5	1.5	1.2	0.6	0.8	0.4	0.7	1.1
6	159.0	25.1	2.3	0.4	31.1	2.3	8.2	15.5	1.1	1.7	3.2	1.5	1.6	0.4	1.0	1.0	2.8	0.8
		171	342	513	684	855	1026	1197	1368	1539	1710	1881	2052	2223	2394	2565	2736	2907
1	173.6	24.9	8.8	9.2	21.5	11.1	7.5	1.7	0.9	0.6	0.6	1.1	0.3	0.3	0.6	1.0	2.8	1.4
4	343.9	20.5	2.4	4.8	18.6	4.0	30.2	5.2	4.8	2.9	1.4	0.5	0.8	0.4	0.5	0.3	1.1	1.6
6	84.6	6.4	6.3	13.9	7.2	8.9	16.9	2.7	4.1	1.4	1.5	2.7	2.4	2.5	6.5	10.2	5.6	0.8

[1] Where the percentages do not sum up exactly to 100 it is because the computation included upper partials higher than those printed. In no cases are the omitted figures large enough to affect the statement of facts.

THE VOWEL *a°*, *continued.*

		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.	XVII.
		192	384	576	768	960	1152	1344	1536	1728	1920	2112	2304	2496	2688	2880	3072	3264
1	261.0	19.9	6.9	33.6	18.7	10.0	1.5	0.4	0.8	0.8	1.2	0.4	0.4	0.8	0.8	1.5	0.8	1.5
4	295.9	18.6	3.4	5.7	18.9	10.3	28.8	2.6	3.7	0.5	2.0	0.5	0.8	0.2	0.4	1.1	1.7	0.8
6	154.4	23.4	9.1	4.5	10.9	6.2	14.2	1.7	3.8	1.7	6.9	0.6	0.9	2.7	5.1	3.8	2.0	2.5
7	39.0	34.6	8.7	0.5	9.2	2.6	28.7	1.0	3.8	2.1	0.6	2.0	1.0	1.3	0.6	0.3	1.5	1.5
		227	454	681	908	1135	1362	1589	1816	2043	2270	2497	2724	2951	3178	3405	3632	3859
1	524.0	20.4	9.4	24.7	26.7	4.0	4.6	0.8	0.8	0.8	0.8	1.4	1.5	1.2	1.5	0.4	0.4	0.6
4	307.5	23.7	8.7	33.3	12.1	9.2	3.4	1.2	2.1	0.4	0.2	0.7	1.7	0.4	1.1	0.9	0.7	0.2
6	157.2	20.2	8.6	21.6	17.1	4.6	4.6	0.1	1.8	0.5	4.5	4.5	3.2	2.9	3.2	0.5	0.5	0.6
7	63.1	30.3	7.1	15.1	9.4	4.4	4.6	2.4	2.4	2.2	1.4	4.6	8.3	3.5	2.7	0.8	0.2	0.6
		256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	4352
1	345.9	11.6	18.2	12.6	37.0	4.6	1.3	3.3	3.9	1.6	0.9	0.4	1.3	0.7	0.7	0.7	0.6	0.6
4	203.5	11.9	9.0	32.6	22.0	8.3	4.0	1.3	1.0	1.3	0.4	0.5	2.4	1.4	1.4	0.8	0.9	0.8
6	142.0	14.2	9.8	12.8	39.4	3.0	4.5	1.7	1.5	2.8	6.1	0.7	0.4	0.7	1.2	0.1	0.7	0.4
7	102.2	14.3	11.9	10.1	37.8	5.8	4.2	2.8	1.7	1.3	2.9	2.7	1.6	0.7	0.5	0.3	0.7	0.7
		288	576	864	1152	1440	1728	2016	2304	2592	2880	3168	3456	3744	4032	4320	4608	4896
7	95.8	9.7	39.8	10.2	9.0	7.9	3.4	2.6	1.6	3.7	4.0	2.2	2.9	0.5	0.8	0.4	0.8	0.5
11	82.9	15.2	9.2	24.2	12.3	16.5	3.3	3.3	1.6	2.0	1.3	1.3	2.0	1.6	2.2	1.0	1.6	1.4
		320	640	960	1280	1600	1920	2240	2560	2880	3200	3520	3840	4160	4480	4800	5120	5440
5	49.2	24.5	24.3	17.0	8.1	1.2	3.1	2.8	3.1	3.5	0.6	2.4	0.8	3.2	1.8	0.8	1.0	1.8
7	63.6	3.9	42.0	14.9	6.8	3.3	5.8	3.9	4.9	9.4	0.8	1.4	1.0	0.6	0.2	0.8	0.3	0.0
11	47.2	12.1	31.1	28.0	9.5	5.5	6.8	1.5	1.3	0.4	0.9	1.1	0.2	0.6	0.4	0.4	0.0	0.2
		342	684	1026	1368	1710	2052	2394	2736	3078	3420	3762	4104	4446	4788	5130	5472	5814
7	213.9	4.5	20.2	50.3	2.4	3.9	0.8	1.1	8.4	1.3	0.7	0.4	0.3	1.3	1.3	0.8	1.2	1.1
11	104.5	6.8	15.3	30.3	25.4	4.6	5.5	4.3	1.9	0.3	1.0	0.9	0.4	0.5	0.5	0.8	0.8	0.7
		384	768	1152	1536	1920	2304	2688	3072	3456	3840	4224	4608	4992	5376	5760	6144	6528
5	152.5	3.0	22.5	44.5	25.5	0.9	0.3	2.1	1.2									
7	93.5	7.2	25.1	32.8	3.2	6.1	1.5	3.2	7.3	1.0	0.7	2.4	1.3	1.3	1.7	1.8	1.8	1.6
11	311.3	6.8	18.1	51.4	17.9	0.7	2.0	2.1	1.0									
		454	908	1362	1816	2270	2724	3178	3632									
7	177.9	11.0	55.9	8.5	2.5	5.5	14.2	0.7	1.7									
11	252.0	8.0	56.3	21.9	2.1	1.6	1.9	5.4	2.8									
		512	1024	1536	2048	2560	3072	3584	4096									
5	121.7	4.9	60.2	18.5	5.2	1.8	6.5	0.8	2.0									
7	81.5	12.8	62.8	12.6	3.1	2.6	4.2	1.1	0.8									
11	276.1	10.2	49.8	29.2	4.7	2.3	1.3	1.5	1.0									

The Vowel *O*.

		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	XI.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.	XVII.
		113	226	339	452	565	678	791	904	1017	1130	1243	1356	1469	1582	1695	1808	1921
1	55.0	42.2	11.5	10.2	2.8	12.7	0.9	1.6	0.8	0.8	1.3	1.1	1.3	1.8	1.8	1.1	0.8	1.1
6	89.2	15.3	39.8	9.2	15.3	2.6	7.6	1.0	1.2	0.8	0.8	1.0	1.0	0.2	0.8	0.6	0.3	0.3
		128	256	384	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048	2176
1	66.9	27.5	21.0	8.2	6.7	23.2	2.3	0.6	1.0	0.1	0.5	0.6	0.4	0.1	0.8	0.9	0.8	0.1
4	104.0	12.4	24.4	5.5	11.6	11.9	10.1	7.4	3.8	2.4	1.4	1.5	0.9	1.1	0.1	0.4	0.1	0.6
		144	288	432	576	720	864	1008	1152	1296	1440	1584	1728	1872	2016	2160	2304	2448
1	132.0	53.3	20.8	11.3	4.2	2.5	0.5	0.2	0.2	0.5	0.5	0.6	0.1	0.5	0.5	0.2	0.5	0.5
6	249.7	17.7	4.2	11.4	20.6	0.4	7.8	15.6	1.8	3.8	0.8	2.1	1.4	1.3	1.0	1.7	2.7	0.8
		160	320	480	640	800	960	1120	1280	1440	1600	1760	1920	2080	2240	2400	2560	2720
1	145.1	69.3	9.2	5.4	5.0	3.1	0.6	0.7	0.4	0.6	0.3	0.8	0.6	0.6	0.1	0.5	0.4	0.6
4	257.3	34.7	7.9	6.1	22.2	3.3	9.2	5.4	1.9	0.6	1.2	0.6	0.3	0.8	0.5	1.0	0.2	0.1
		171	342	513	684	855	1026	1197	1368	1539	1710	1881	2052	2223	2394	2565	2736	2907
1	152.0	18.5	25.7	11.1	25.6	1.6	2.5	1.0	1.0	0.7	0.3	0.6	0.7	0.6	1.4	0.9	1.6	1.8
4	164.5	28.6	5.8	17.4	9.2	9.0	12.4	4.5	1.4	1.4	0.7	0.8	0.7	0.8	0.4	0.7	0.9	1.7
6	357.1	12.9	7.9	17.2	6.7	8.5	28.2	0.8	1.9	0.2	1.9	1.6	3.0	3.2	1.3	1.5	1.5	1.7
		192	384	576	768	960	1152	1344	1536	1728	1920	2112	2304	2496	2688	2880	3072	3264
1	143.0	30.8	17.6	12.0	19.9	2.0	2.5	1.5	1.0	2.1	0.8	0.2	0.4	0.7	1.2	1.2	1.0	2.7
4	261.2	14.4	4.0	18.9	11.8	11.6	19.2	1.8	1.8	3.1	1.4	1.1	0.7	0.4	0.9	2.5	4.2	2.2
6	178.3	20.0	6.3	30.1	5.2	18.7	4.8	3.2	1.8	2.0	1.9	0.8	0.1	2.1	1.7	0.7	0.4	0.2
		227	454	681	908	1135	1362	1589	1816	2043	2270	2497	2724	2951	3178	3405	3632	3859
4	221.5	44.9	12.0	23.2	5.4	5.0	1.5	1.4	1.8	0.3	0.8	0.5	0.6	0.5	0.9	0.3	0.8	0.1
5	60.6	43.1	9.1	17.0	6.8	12.7	3.4	0.5	0.8	0.8	1.3	0.2	0.6	1.5	0.4	1.1	0.5	0.2
6	283.0	15.1	15.8	15.5	28.7	5.6	6.7	1.4	1.5	2.1	3.5	0.8	0.4	0.7	1.0	0.9	0.1	0.2
		256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	4352
1	221.7	13.3	22.0	3.1	49.0	4.2	1.7	1.2	1.2	0.5	1.3	0.4	0.1	1.3	0.2	0.3	0.1	0.1
4	288.1	27.6	31.1	16.9	9.8	3.4	2.9	0.7	0.1	0.6	0.7	1.6	1.1	1.4	1.1	0.5	0.3	0.2
6	339.6	10.5	14.8	8.2	45.5	3.0	5.1	1.1	1.9	0.9	2.9	0.8	1.4	0.7	1.9	0.6	0.4	0.3
		288	576	864	1152	1440	1728	2016	2304	2592	2880	3168	3456	3744	4032	4320	4608	4896
5	47.0	30.4	20.6	12.3	9.2	8.9	4.9	3.8	2.6	0.9	0.6	1.5	1.1	0.9	0.9	0.4	0.4	0.6
7	42.0	16.9	18.8	11.0	16.4	8.6	1.6	3.1	2.6	1.9	9.1	2.4	3.1	1.4	1.2	0.7	0.5	0.7
		320	640	960	1280	1600	1920	2240	2560	2880	3200	3520	3840	4160	4480	4800	5120	5440
5	42.6	39.8	15.6	5.5	21.9	2.3	2.3	2.1	2.3	0.9	0.9	1.6	0.9	0.9	0.9	1.4	0.5	0.2
7	107.7	7.4	46.8	11.4	8.0	5.4	4.8	4.1	3.3	4.8	1.6	0.5	0.3	0.4	0.3	0.6	0.1	0.2
11	85.6	13.6	40.1	17.8	16.9	4.8	2.2	1.1	0.4	0.5	0.9	0.4	0.3	0.2	0.2	0.1	0.1	0.4
		342	684	1026	1368	1710	2052	2394	2736									
5	238.6	4.3	26.4	47.4	18.7	0.7	1.0	0.9	0.6									
11	233.6	4.0	25.3	50.0	13.8	2.2	1.6	1.3	1.8									
		384	768	1152	1536	1920	2304	2688	3072	3456	3840	4224	4608	4992	5376	5760	6144	6528
5	134.4	10.1	51.6	18.5	11.7	2.4	1.6	0.8	0.9	0.2	0.4	0.2	0.1	0.4	0.3	0.4	0.3	0.1
7	119.3	3.0	44.7	11.0	8.6	6.8	5.0	4.4	0.6	9.5	1.5	1.9	1.2	0.4	0.6	0.3	0.3	0.2
11	174.2	4.9	29.4	45.2	16.2	1.4	1.0	0.5	1.4									

THE VOWEL *O*, *continued*.

		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.	XVII.
		454	908	1362	1816	2270	2724	3178	3632									
5	121.2	17.1	62.7	12.3	2.8	1.8	2.0	0.7	0.6									
11	104.3	15.0	50.1	25.8	1.2	1.0	2.4	3.2	1.3									
7	382.6	18.2	52.5	9.6	2.1	2.5	11.1	2.7	1.3									
		512	1024	1536	2048	2560	3072	3584	4096	4608	5120	5632	6144	6656	7168	7680	8192	8704
5	116.4	11.6	33.6	32.1	10.3	4.4	1.4	0.4	2.1	6.7	0.9	0.5	0.2	0.7	0.1	0.1	3.3	0.5
7	155.6	23.7	62.7	6.5	3.1	1.2	1.5	0.6	0.7									
11	107.1	6.4	53.1	20.3	11.8	3.3	2.4	1.6	1.1									

THE VOWEL *U*.

		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIX.	XX.	XXI.	XXII.
		113	226	339	452	565	678	791	904	1017	1130	1243	1356	1469	1582	1695	1808	1921
1	130.6	17.1	41.8	8.6	1.6	3.7	7.6	1.4	1.0	2.1	0.4	2.4	1.4	1.7	0.8	0.5	0.5	0.6
4	109.3	8.1	52.6	12.7	5.7	3.3	3.4	0.8	1.8	0.5	0.4	1.2	1.5	0.7	0.7	0.6	1.8	1.2
		128	256	384	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048	2176
1	124.8	37.6	28.6	5.2	2.4	6.5	4.0	0.4	1.9	0.4	0.6	0.6	0.6	1.4	0.8	0.2	0.6	0.9
4	26.7	3.0	39.1	10.0	9.0	3.7	2.7	3.4	2.6	0.8	1.5	3.4	1.9	4.9	1.9	2.2	1.5	1.2
		144	288	432	576	720	864	1008	1152	1296	1440	1584	1728	1872	2016	2160	2304	2448
1	274.3	63.6	16.7	2.0	2.9	1.9	3.6	1.6	0.4	1.1	0.4	0.4	0.0	0.4	0.3	0.8	0.3	0.7
4	225.3	54.7	19.0	8.7	1.6	2.6	3.5	1.8	0.9	0.1	0.7	1.9	0.9	1.0	0.4	0.2	0.2	1.0
6	312.9	59.1	18.1	7.2	0.9	1.4	3.6	2.2	0.4	0.5	1.1	0.9	0.3	0.5	1.0	1.3	0.5	0.6
		160	320	480	640	800	960	1120	1280	1440	1600	1760	1920	2080	2240	2400	2560	2720
1	216.1	45.3	23.0	2.8	14.6	2.5	1.7	1.4	0.4	0.1	0.8	0.6	0.6	0.5	1.0	2.3	1.0	1.4
6	292.3	64.8	18.0	4.4	2.6	0.2	2.0	2.6	0.5	1.1	0.3	0.5	0.1	0.2	0.5	0.9	0.2	0.5
		171	342	513	684	855	1026	1197	1368	1539	1710	1881	2052	2223	2394	2565	2736	2907
1	246.8	32.4	24.0	5.9	19.0	7.6	5.4	0.9	0.9	0.9	0.4	0.8	0.2	1.7	4.6	0.4	1.9	1.6
4	304.5	44.2	20.5	6.6	8.0	9.1	3.3	1.9	1.1	1.2	0.9	0.3	0.4	0.1	0.4	0.8	0.6	0.6
6	300.4	35.2	21.9	6.5	10.0	5.8	8.7	2.4	0.0	1.3	0.4	0.6	2.6	2.1	1.3	0.3	0.3	0.6
		192	384	576	768	960	1152	1344	1536	1728	1920	2112	2304	2496	2688	2880	3072	3264
1	347.0	34.0	28.8	4.5	14.8	2.9	1.7	1.6	0.5	2.3	0.5	1.1	1.1	3.4	0.4	1.2	0.8	0.4
4	185.9	65.3	14.4	4.4	4.9	2.9	0.8	1.5	0.4	0.5	0.4	0.6	1.0	0.3	0.4	0.5	0.4	0.4
6	330.5	30.5	21.5	5.6	6.6	1.9	18.8	1.0	1.8	0.5	1.4	2.0	3.0	1.0	0.6	1.5	0.7	1.6
		227	454	681	908	1135	1362	1589	1816	2043	2270	2497	2724	2951	3178	3405	3632	3859
1	554.0	74.7	3.6	14.3	1.3	1.4	0.5	1.1	0.3	0.4	0.6	0.2	0.3	0.1	0.2	0.7	0.1	0.2
4	442.6	70.4	7.1	8.2	5.9	1.5	1.3	0.3	0.8	0.9	0.5	1.0	0.2	0.2	0.3	0.2	0.3	0.9
6	197.5	54.8	22.2	7.0	2.9	2.2	1.0	1.0	2.1	4.1	0.8	0.3	0.1	0.2	0.4	0.5	0.3	0.1

The Vowel *U*, *continued.*

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XVI.	XV.	XVI.	XVII.
		256	512	768	1024	1280	1536	1792	2048	2304	2560	2816	3072	3328	3584	3840	4096	4352
1	455.0	41.8	13.5	19.3	2.5	2.1	2.7	0.8	1.1	5.0	1.3	4.3	2.0	1.3	0.5	0.5	0.2	1.3
4	191.1	66.9	8.6	12.9	3.9	1.7	0.9	1.1	0.4	0.4	0.7	0.3	0.3	0.3	0.9	0.4	0.2	0.3
6	181.9	56.5	14.7	17.2	3.6	1.5	1.4	0.7	1.1	0.6	0.6	0.7	0.3	0.2	0.4	0.1	0.1	0.3
5	72.1	74.7	0.8	4.2	3.0	0.7	0.5	1.7	0.1	1.7	1.4	3.2	2.2	1.9	1.5	1.2	0.5	0.7
		288	576	864	1152	1440	1728	2016	2304	2592	2880	3168	3456	3744	4032	4320	4608	4896
7	47.7	22.9	13.0	5.7	5.6	5.4	4.2	3.8	2.5	6.0	6.1	14.9	4.8	0.6	1.7	0.2	1.1	1.5
		320	640	960	1280	1600	1920	2240	2560	2880	3200	3520	3840	4160	4480	4800	5120	5440
11	49.2	34.9	39.2	8.9	4.0	5.6	1.8	0.4	0.8	0.6	0.8	0.4	0.2	1.0	0.8	0.2	0.2	0.2
		342	684	1026	1368	1710	2052	2394	2736	3078	3420	3762	4104	4446	4788	5130	5472	5814
5	121.9	29.5	5.3	49.2	2.4	4.4	3.5	1.1	4.6									
7	58.7	13.8	14.8	24.3	6.5	6.3	1.5	1.9	15.3	8.5	1.4	1.0	0.7	1.0	0.7	1.2	0.7	0.4
11	54.5	16.0	24.2	51.5	3.1	2.2	0.4	1.3	1.3									
		384	768	1152	1536	1920	2304	2688	3072	3456	3840	4224	4608	4992	5376	5760	6144	6528
5	103.7	30.1	2.0	11.3	38.5	4.8	2.2	2.4	2.3	1.0	0.8	0.5	0.7	1.0	0.7	0.8	0.3	0.6
7	70.6	21.1	35.6	11.3	5.2	4.8	2.4	2.0	1.7	4.5	3.5	3.4	1.1	1.4	0.1	0.3	0.8	0.8
11	46.8	27.2	36.3	28.4	2.1	1.5	2.1	0.9	1.5									
		454	908	1362	1816	2270	2724	3178	3632	4086	4540	4994	5448	5902	6356	6810	7264	7718
5	243.9	39.4	45.0	9.1	0.4	2.3	2.7	0.2	0.9									
7	65.4	19.4	29.7	12.3	4.3	4.4	15.6	6.6	1.1	1.9	0.6	0.9	0.6	0.6	0.5	0.5	0.5	0.5
11	82.8	29.7	45.8	15.5	5.7	1.4	0.6	0.9	0.4									
		512	1024	1536	2048	2560	3072	3584	4096	4608	5120	5632	6144	6656	7168	7680	8192	8704
7	124.8	28.2	32.1	19.6	3.4	3.9	7.1	1.5	0.6	0.4	0.6	0.5	0.4	0.4	0.3	0.4	0.4	0.2
7	47.0	21.7	19.2	18.1	4.7	5.9	12.1	7.9	3.2	1.1	1.2	0.2	1.1	0.2	0.9	1.0	0.6	0.9

THE MEASUREMENT OF INDUCTANCE AND CAPACITY BY MEANS OF THE DIFFERENTIAL BALLISTIC GALVANOMETER.

BY WILLIAM J. RAYMOND.

THE differential galvanometer has long been used in the comparison of electric resistances, but apparently has not been employed to any great extent in the measurement of inductance and capacity. Believing that it deserves a more prominent place in electrical laboratories than has yet been given to it, the following descriptions of a number of methods for the determination of self-inductance, mutual inductance and capacity have been prepared. Some of these are null methods, others require the measurement of ballistic deflections of the needle of the galvanometer. Including three methods previously published by Kohlrausch and Niven, all have been tested by the present writer. He has not discovered in the literature of the subject any other methods which make use of the differential galvanometer.

The differential telephone[1] and differential transformer[2] have recently been employed in the measurement of inductance and capacity with great precision in the expression of results. The methods described are necessarily null methods, and make use of a system of conductors analogous to the Wheatstone quadrilateral, or a somewhat simpler, parallel arrangement of branches containing the coils of the telephone or transformer. The double condition of equality of amplitude and phase of alternating currents in the differential coils is required for silence in the telephone. Possibly in some of the null methods described in the present paper, the telephone may replace the galvanometer with gain in precision of measurements. Apparently in most of the methods the double condition can not be reached.

[1] Ho, Electrical World, 41, p. 884, 1903; Ho, PHYS. REV., XIX., p. 166, 1904; Duane and Lory, PHYS. REV., XVIII., p. 275, 1904.

[2] Trowbridge, PHYS. REV., XX., p. 65, 1905.

In laboratories and testing rooms reasonably free from magnetic disturbances, a galvanometer of the form subsequently described may be employed. In localities where there are fluctuations in magnetic field, a galvanometer of the D'Arsonval type, differentially wound, may be found practicable. A differential galvanometer of this type was described at the meeting of the American Physical Society, held at St. Louis in September, 1904.[1] It appears that most if not all of the methods of measurement given in the present paper may be carried out with such a galvanometer, designed for ballistic use. The damping factor for the vibration of the coil should be made as small as possible, and its period should be rather long.

Working formulas given in this paper are easily deduced from equations of electromotive force, as exemplified in method 1. In nearly all cases current divides between parallel branches connecting points A and B. Expressions for difference of potential by way of the branches between A and B, during the period of variable currents, are equated, and integrated over this period. This leads to an equation connecting inductances or capacities, resistances, quantities of electricity passing through the coils of the galvanometer, and final, steady currents. Equality of final currents permits the cancelation of equal terms, and in the null methods the condition of equality of quantities passing through the galvanometer is introduced. In deflection methods difference between quantities is written as proportional to the deflection of the needle of the galvanometer. The proportional factor need not be known since it disappears by cancelation from ratios of deflections. When there are more than two branch circuits, ratios of steady currents are expressed as ratios of resistances. The charge of a condenser is written as capacity multiplied by difference of potenial the latter factor appearing as current multiplied by resistance, when resistance is one of the parallel branches connects the terminals of the condenser.

Accuracy of results obtainable by the methods described in this paper rests upon a number of factors, some of which are apparent from inspection of working formulas. The writer was more con-

[1] J. C. Shedd, PHYS. REV., XIX., p. 301 (abstract).

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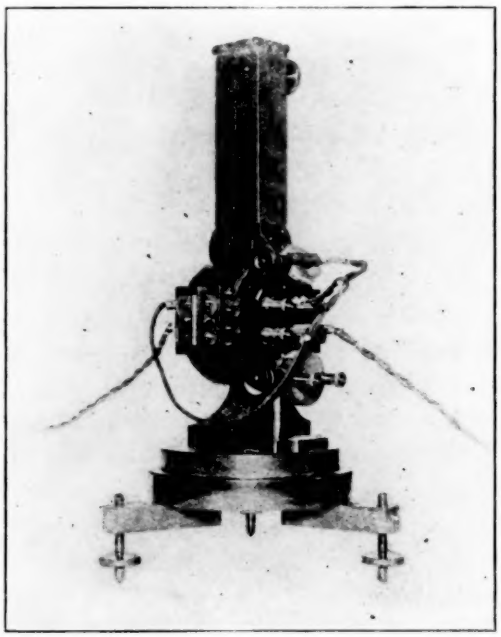
cerned to survey the whole field than to investigate the accuracy which may be obtained by any one method. Yet in this general survey, among separate measured values of the same quantity, agreement within two or three tenths of one per cent. was reached, not only in one method but among several. It appears that with care agreement within one tenth per cent. may be easily reached, especially among the null methods. With closer attention to detail, still better results may be expected. The question of accuracy of standards is not touched upon here, but merely the precision of measurements, that is, the agreement among individual results.

DESCRIPTION OF GALVANOMETER.

The galvanometer used in this work was constructed by Mr. Stamper, the mechanician in the department of physics of the University of California. Two coils are contained, one in the upright standard of the instrument, one in a hinged portion, so arranged as to permit free access to the needle and suspending fiber. Each coil was wound by laying two wires side by side, and keeping them together from beginning to end. This insured nearly equal values of magnetic effect, of self-inductance and of resistance of the windings. When the four windings, constituted as described, and numbered 1 and 2 in the hinged part of the galvanometer, and 3 and 4 in the fixed standard, were connected in series, 1 and 3 opposing 2 and 4, the deflection of the needle was 12 minutes of arc when the current flowing was 0.15 ampere. It was considered desirable to secure more exact magnetic balance. To this end a small accessory coil of three turns of wire was mounted at the back of the galvanometer and connected in series with windings 1 and 3. This coil turns on a horizontal axis and is easily adjusted so that there is no deflection of the needle when the four windings are connected as previously described and traversed by steady current many times greater than is used in the measurement of inductance or capacity. The coils are connected externally by double conductors so that possible displacement of wires during the progress of a measurement may have no effect upon the magnetic balance. The self-inductances of the coils of the galvanometer are assumed equal in the deduction of working formulas. In the instrument described

they were found to be close enough in value for the immediate purpose of the investigation, without further adjustment. If necessary a pair of small coils, mounted according to the plan of the Ayrton and Perry variable standard of inductance, may be placed at a safe distance from the needle of the galvanometer and connected in series with the coil having the smaller self-inductance. The resistance of the four windings of the galvanometer, connected in series, is 12.9 ohms at 16° C. Adjustment of the branch circuits to equality of resistances is reached by the plan described under the first of the following methods of measurement.



The suspended system or needle of the galvanometer consists of an ellipsoidal magnet, 6.5 mm. in diameter, mounted on a sliver of bamboo. A small mirror is carried at the upper end of the bamboo stem and just below the mirror is a thin brass disk, 16 mm. in diameter, turned with a heavy rim to secure large moment of inertia with small mass. The whole is hung from a silk fiber. The purpose of the brass disk is to increase the period of vibration of the needle, for convenience in reading turning points on the telescope-

scale in the deflection methods. Ballistic deflection of the needle for given magnetic impulse of current passing is thereby reduced in the same proportion as the period is increased, so that in practice one has to strike a balance between convenience of use and sensitiveness. To bring the needle to rest rapidly and effectively, just before a ballistic deflection is to be measured, a second accessory coil of 3.7 cm. diameter and 15 turns of wire is mounted back of the instrument and made part of an independent circuit through a battery cell and a double key, placed within reach of the observer's hand. One key closes the circuit through 150 ohms resistance mounted on the base of the key; the second key shunts out this resistance when a stronger, temporary, controlling field is desired at the needle. To vary the controlling field further the plane of the accessory coil may be turned about a horizontal axis. Thus when the needle is vibrating through large angles the stronger field, applied at the proper time, rapidly reduces the swing. When the needle is nearly at rest a weaker field is desirable.

When the most favorable conditions for securing accuracy of results are considered, it must be at once evident that the galvanometer here described is adapted to a limited range of measurements, because its resistance is small. Notably in the measurement of capacities and of inductances associated with large resistances, a more sensitive galvanometer would be required. In neither of the cases cited would increased resistance of the galvanometer, due to the use of a greater number of turns of finer wire, be objectionable, while the deflection of the needle when there is a given difference between the quantities of electricity passing through the two coils, would be greatly increased. If necessary, further increase of sensitiveness may be reached by known methods, such for example as the use of an astatic needle.

METHODS OF MEASUREMENT.

All of the methods to be described are essentially comparisons between any two of the four quantities, resistance, self-inductance, mutual inductance and capacity. Incidentally measurements of period of vibration of the galvanometer needle and of ballistic deflection are involved. About half of the methods are null

methods which seek for adjustment securing balance between impulses of opposing force-moments exerted upon the needle, the impulses integrated over the period of variable current, and both impulses of extremely short duration. Since four quantities may be combined, two and two, in ten different ways, a convenient classification of methods is suggested. These classes are the comparison of two resistances; two self-inductances; two mutual inductances; two capacities; resistance and self-inductance; resistance and mutual inductance; resistance and capacity; self-inductance and mutual inductance; self-inductance and capacity; mutual inductance and capacity. Comparison of resistances with the differential galvanometer, using steady currents, is so familiar that no further reference will here be made to it. The methods described in this paper will then fall into the remaining nine classes. They are doubtless not all which might be devised, but will serve as illustrative examples, covering fairly well the usual range of measurements.

When dealing with inductive resistances, the usual precautions are to be observed. Coils must be sufficiently separated to avoid mutual magnetic effect, and their positions and distances should be such that no effect is perceivable at the galvanometer. A reversing key or commutator is placed in the battery circuit, instead of a simple key merely to open or close the circuit. In this connection Lord Rayleigh[1] remarks that: "A reversal has two advantages over a simple make or break. In the first place the effect is doubled and is therefore more easily measured; and in the second the battery is more likely to work in a uniform manner, the circuit being always closed except for a fraction of a second at the instant of reversal." It may be added that if the damping of the needle differs sensibly between open and closed circuit through the galvanometer, the correction for it will be considerably simplified.

Description of Methods.

Method 1, Comparison of Self-Inductances.—(Fig. 1.) A battery cell and commutator K are arranged to send current from A to B through a divided circuit. On one side are the inductive resistance

[1] Phil. Trans. R. S., 1882, Part II., p. 678.

L_1 (or L_2), a box of non-inductive resistance coils, and windings 1 and 3 of the galvanometer, represented in the figure by a circle; on the other side a resistance box, and windings 2 and 4 of the galvanometer, represented by the second circle. B is a contact key which may be moved at will along a german silver wire CD, 50 centimeters in length, to secure adjustment to exact equality of the total resistances, R_1 and R_2, on the two sides. Resistance may be included in the battery branch if necessary for ballast. (a) With inductance L_1 in circuit, balance the resistances for zero deflection of the galvanometer needle, using steady current. Then R_1 is equal to R_2. (b) Bring the needle of the galvanometer to rest, reverse the commutator K and measure the ballistic deflection d_1. (c) Insert the inductance L_2 in place of L_1 and adjust the resistance in series with it to balance the same resistance R_2 previously used. (d) Reverse the commutator as before and measure the ballistic deflection d_2. The ratio of inductances is then

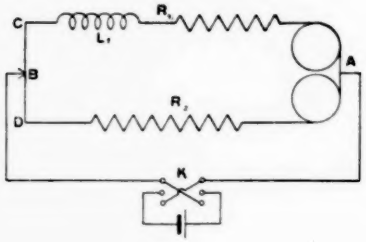


Fig. 1.

$$\frac{L_1}{L_2} = \frac{d_1}{d_2}. \tag{1}$$

To deduce this expression, write the difference of potential between A and B for any time during the period of variable currents,

$$R_1 i_1 + L_1 \frac{di_1}{dt} + I' \frac{di_1}{dt} - M \frac{di_2}{dt} = R_2 i_2 + I' \frac{di_2}{dt} - M \frac{di_1}{dt}, \tag{2}$$

in which I' is the self-inductance of galvanometer windings 1 and 3, or 2 and 4, M is their mutual inductance, and i with indicating subscript is current intensity. Integrating over the period of variable currents,

$$R_1 Q_1 + L_1 I_1 + I' I_1 - M I_2 = R_2 Q_2 + I' I_2 - M I_1, \tag{3}$$

in which I_1 and I_2 are final, equal, steady currents, and Q_1 and Q_2 are quantities of electricity passing through the coils of the galva-

nometer during the period of variable flow. Canceling equal terms and dropping unnecessary subscripts,

$$Q_2 - Q_1 = \frac{L_1 I}{R}. \tag{4}$$

Similarly, from operations (*c*) and (*d*),

$$Q_2' - Q_1' = \frac{L_2 I}{R}. \tag{5}$$

The differential quantities are proportional to the ballistic deflections d_1 and d_2, the proportional factor involving the period of vibration of the needle of the galvanometer and constants of the coils. The deflections need not be corrected for damping, and are measured either in radians or directly in centimeters on a scale, provided that the telescope remains at a fixed distance from the galvanometer.

$$\frac{Q_2 - Q_1}{Q_2' - Q_1'} = \frac{d_1}{d_2} = \frac{L_1}{L_2}. \tag{6}$$

Method 2, Comparison of Self-Inductances. — This is a null method due to Professor C. Niven.[1] Coils whose inductances are L_1 and L_2 (L_1 greater than L_2), are joined in series with non-inductive resistances and connected one on each side of the galvanometer circuit. Referring to the similar arrangement of Fig. 2, L_1 is placed nearer the point C than its series resistance, L_2 nearer the galvanometer, and R_2 is omitted. If each non-inductive resistance is made equal to the resistance of the opposing coil, there will be no permanent deflection of the needle of the galvanometer when steady current is applied. Further, points between the coils and the series resistances are at the same potential, and may be joined by a cross-wire without disturbing balance for steady currents. If the resistance of this cross-connection is adjustable, a value S will be found by trial, such that there is no ballistic deflection of the needle of the galvanometer, when current is reversed by means of the commutator K. If R_1 is the resistance between point B and the connection joining L_1 and its series resistance, the ratio of inductances is

$$\frac{L_1}{L_2} = \frac{2R_1 + S}{S}. \tag{7}$$

[1] Niven, Phil. Mag. (5), Vol. 24, p. 234, 1887; Gray, Abs. Meas. in El. and Mag., Vol. 2, p. 463, 1893.

Method 3, Comparison of Self-Inductances.—This is a null method for which the scheme of connections is indicated in Fig. 2. If the resistance of one of the coils, R_1, is considerably greater than that of the other, the non-inductive resistance in series with R_1 may be omitted. The resistances are first to be balanced for steady currents; then adjusted for zero ballistic deflection when commutator K is reversed. Since balance for steady currents is to be maintained, this will require double adjustment of resistance R_2, in parallel with R_1, and one of the series resistances. With inductances of ordinary magnitude, it will be found that near the final adjustment of R_2 small changes in series resistances will suffice to maintain balance for steady currents. These changes may be made by shifting contact B along the german silver wire CD. If there is difficulty in deciding upon the best value of R_2 for zero ballistic deflection, the usual method of interpolation between values giving small right and left deflections may be employed. The ratio of inductances is

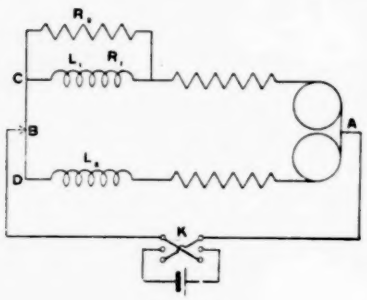


Fig. 2.

$$\frac{L_1}{L_2} = \frac{(R_1 + R_2)^2}{R_2^2}. \qquad (8)$$

This equation may be easily deduced from an equation of electromotive forces by the plan followed in the establishment of equation (1).

Method 4, Comparison of Mutual Inductances, Null Method.—(Fig. 3.) Four coils, whose self-inductances are denoted by L_1, L_2, L_3 and L_4, and mutual inductances by M_{12} and M_{34}, are connected, together with non-inductive resistances and the differential galvanometer, as indicated in the figure. (*a*) Balance the resistances for zero deflection of the needle of the galvanometer, using steady currents. (*b*) Connect the coils so that the magnetic fields of coils 3 and 4 shall coincide in direction, while the fields of coils 1 and 2 are opposed when current flows from A to B. Adjust the resistances in series with coils 1 and 4 so that there may be no ballistic deflec-

tion when the commutator K is reversed. (*c*) Connect the coils so that the magnetic fields of both pairs shall be opposed in direction, and again adjust for zero ballistic deflection. Interpolation may be employed to obtain the best values of resistances. If R_1 denotes the total resistance from A to B by way of coil 1, and R_4 denotes the total resistance by way of coil 4 in adjustment (*b*); and R_1' and R_4' denote corresponding values in adjustment (*c*); then the ratio of mutual inductances is

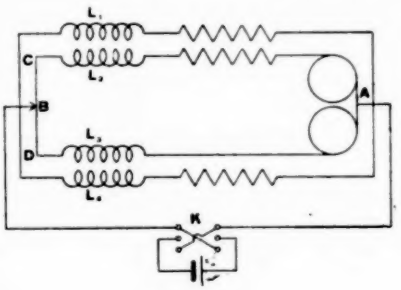


Fig. 3.

$$\frac{M_{12}}{M_{34}} = \frac{R_1 R_1' (R_4 + R_4')}{R_4 R_4' (R_1 - R_1')}. \quad (9)$$

With other magnitudes of mutual inductances and self-inductances than those employed by the writer, other connections than those described under (*b*) and (*c*) may be required. Corresponding changes in the signs of the working formula will then be necessary.

Method 5, Comparison of Mutual Inductances, Deflection Method.— Connect coils 1 and 2, whose mutual inductance is M_{12}, in series on one side of the divided circuit of the galvanometer; and coils 3 and 4, whose mutual inductance is M_{34}, on the other side. (*a*) Insert non-inductive resistance in the branch of the circuit having the lesser total resistance, and balance for steady currents. (*b*) With the magnetic fields of both pairs of coils coinciding in direction when current is flowing, reverse the commutator K and measure the ballistic deflection d_1. (*c*) Reverse the terminals of coil 3, so that its field may oppose that of coil 4, and measure the ballistic deflection d_2, obtained by reversal of the commutator. (*d*) Interchange the terminals of coil 1 so that its field may oppose that of coil 2, and measure the ballistic deflection d_3, obtained by reversal of the commutator. The required ratio of mutual inductances is

$$\frac{M_{12}}{M_{34}} = \frac{d_2 - d_3}{d_2 - d_1}. \quad (10)$$

The signs in this equation depend upon the order in which the terminals are interchanged in operations (*b*), (*c*) and (*d*).

Method 6, Comparison of Capacities, Null Method. — Equal, non-inductive resistances are connected as indicated in Fig. 4. The sliding contact B is adjusted for final balance of resistances, shown by zero deflection with steady current. Condensers 1 and 2, whose capacities are C_1 and C_2, are joined as indicated, one side of each connected to a traveling plug, so that each condenser may be put in parallel with any required portion of the corresponding resistance. If R_1 and R_2 are the resistances thus bridged between point B and the traveling plugs, when the ballistic deflection of the galvanometer is zero, the ratio of capacities is

$$\frac{C_1}{C_2} = \frac{R_2^2}{R_1^2}. \tag{11}$$

Method 7, Comparison of Capacities. — This is a null method for which the scheme of connections is indicated in Fig. 5. Condensers

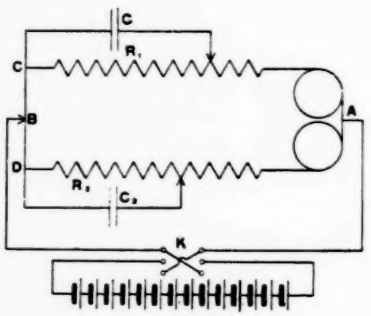


Fig. 4.

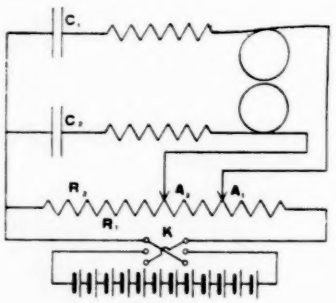


Fig. 5.

1 and 2 are placed as shown with non-inductive resistances in series, to avoid too heavy discharge currents through the coils of the galvanometer. A battery of several cells is connected to a resistance of twenty thousand ohms or more, through a commutator K. Traveling plugs A_1 and A_2 permit the bridging of resistances R_1 and R_2 as shown. By trial the plugs are placed so that the ballistic deflection of the needle of the galvanometer is zero when the commutator is reversed. The ratio of capacities is

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}. \tag{12}$$

Method 8, Comparison of Capacities. — This null method is a modification of the last. Double contact keys are arranged to charge the condensers 1 and 2 when the keys are depressed; and to discharge each condenser through one coil of the galvanometer when they are simultaneously released. The lower contact points of the keys are connected, one to each of two traveling plugs A_1 and A_2, placed as in method 7, and the upper contact points are connected to resistances in the branches of the galvanometer circuit. The discharge currents pass through the coils of the galvanometer when the keys are released, unite and pass through a common wire to the plates of the condensers which are not in connection with the keys. These plates are also joined to one end of the high resistance as in method 7. This arrangement avoids in a measure the trouble arising from faulty insulation of either condenser.

Method 9, Comparison of Capacities, Deflection Method. — Referring to Fig. 2, replace each inductive resistance by one of the two condensers whose capacities C_1 and C_2 are to be compared, and omit the shunt resistance R_2. A battery of several cells will in general be required, instead of the single cell shown in the figure. (*a*) Reverse the commutator K and measure the ballistic deflection d_1. (*b*) Remove both condensers from the branches of the circuit, leaving only non-inductive resistances in series with the coils of the galvanometer. Join the condensers in parallel with each other and with one of the resistances R_1. Adjust the other resistance and the sliding contact B to obtain zero deflection when steady current is flowing. The total resistance from A to B by either branch is now R. (*c*) Reverse the commutator K and measure the ballistic deflection d_2. The ratio of capacities is

$$\frac{C_1}{C_2} = \frac{2R^2 d_2 + R_1^2 d_1}{2R^2 d_2 - R_1^2 d_1}. \qquad (13)$$

The same battery is used throughout and its internal resistance is negligible in comparison with R. The signs in the formula assume that the capacity of condenser 1 is greater than that of condenser 2.

Method 10, Self-Inductance Measured in Henrys. — This is a deflection method of comparing self-inductance and resistance, analogous to Maxwell's method as modified and employed by Lord

Rayleigh and Dr. Schuster in their work on the determination of the ohm in absolute measure.[1] The connections of Fig. 1 are used. (*a*) Balance the resistances for zero deflection when steady current is flowing. (*b*) Reverse the commutator K and measure the resulting ballistic deflection d_1. This deflection is to be corrected in the usual way to the value which would have been found, if there had been no damping. (*c*) Change the resistance on one side of the circuit between A and B by a small amount ΔR, and measure the resulting deflection d_2 when steady current is flowing. If R is the total resistance on each side before the change, and T is the measured period of vibration of the needle of the galvanometer, the inductance is

$$L = \frac{\Delta R \cdot R}{R + \Delta R} \cdot \frac{T}{4\pi} \cdot \frac{d_1}{d_2}. \qquad (14)$$

It is of interest to note that the dimensions of inductance (a length) appear in this equation as resistance (velocity) multiplied by time.

Method 11, Self-Inductance Measured in Henrys, Kohlrausch's Deflection Method, Using Induced Currents.[2] — The coil whose self-inductance is to be determined is connected in one branch of the differential galvanometer circuit as shown in Fig. 1, and an equal, non-inductive resistance is included in the other branch. Points A and B are connected to a Weber magnet-inductor, or to the secondary of an induction coil. (*a*) By this means an induced current of short duration is divided between the branches of the galvanometer circuit. The quantity of electricity passing through each branch is the same, but since current grows more slowly on the side having the larger inductance, there is a sharp movement of the galvanometer needle from rest in its position of equilibrium, to another, momentary position of rest. This measured deflection is d_1. (*b*) By sending induced current from the same source as before through half of the windings of the differential galvanometer, connected in series with a large resistance, a ballistic deflection is obtained and measured. This is to be corrected for damping. Its corrected value is d_2. Let R_1 be the resistance of the coil whose

[1] Proc. R. S. London, Vol. 32, p. 116, 1881; Phil. Trans. R. S. London, 1882, Part II., p. 677.

[2] Ann. d. Phys. u. Chem., Vol. 31, p. 594, 1887.

inductance is desired, or of the equal, non-inductive ballast; R_2 the resistance of each half of the galvanometer; R_3 the resistance of the source of induced current; R_4 the large resistance included in part (*b*); T the period of vibration of the needle of the galvanometer. The required inductance is

$$L = \frac{d_1}{d_2} \cdot \frac{T}{2\pi} \cdot \frac{(R_1 + R_2)(2R_3 + R_1 + R_2)}{R_4 + R_3 + R_2}. \qquad (15)$$

Method 12, Mutual Inductance Measured in Henrys, Deflection Method. — Two coils are connected in series, first with their magnetic fields coinciding in direction when current flows in a given direction, second with their fields opposed. If L_1, L_2 and M are the self-inductances and mutual inductance of the two coils, the total inductance is $L_1 + L_2 + 2M$ in the first case and $L_1 + L_2 - 2M$ in the second case. Measure these total inductances by method 10, each in henrys. The required mutual inductance is one fourth of the difference between the measured values.

Method 13, Capacity Measured in Farads.— This is a deflection method of comparing capacity and resistance. The condenser, whose capacity C is to be determined, is connected in parallel with non-inductive resistance and introduced into one branch of the galvanometer circuit, in place of capacity C_1 as shown in Fig. 4, C_2 being omitted. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Reverse the commutator K and measure the ballistic deflection resulting. Let this when corrected for damping be d_1. (*c*) Change one of the resistances by a small amount ΔR and measure the permanent deflection d_2, using steady current. Let R be the total resistance from A to B by either branch before the change; R_1 the resistance in parallel with the condenser; and T the period of vibration of the needle of the galvanometer. The required capacity is

$$C = \frac{\Delta R \cdot R}{R_1^2(R + \Delta R)} \cdot \frac{T}{4\pi} \cdot \frac{d_1}{d_2}. \qquad (16)$$

Method 14, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of One of Them, Null Method. — Coils 1 and 2, whose self-inductances are L_1 and L_2 and whose mutual in-

ductance is M, are connected as indicated in Fig. 6. For given direction of current from A to B, the magnetic fields of the coils are to be opposed in direction. (*a*) Balance the resistances for zero deflection, using steady currents. (*b*) Having connected non-inductive resistance in series with coil 2, adjust this resistance for zero ballistic deflection when the commutator K is reversed. If R_1 and R_2 are the total resistances from A to B by way of the branches including coils 1 and 2 respectively, the ratio of the self-inductance of coil 1 to the mutual inductance of both coils is

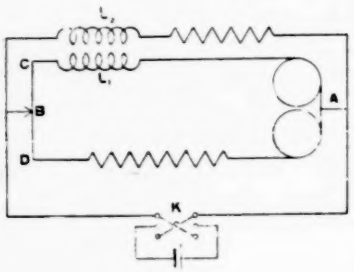


Fig. 6.

$$\frac{L_1}{M} = \frac{R_1}{R_2}. \tag{17}$$

Method 15, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of One of Them, Deflection Method. — The coils, whose self-inductances are L_1 and L_2 and whose mutual inductance is M, are connected in series and included in one branch of the galvanometer circuit, in place of L_1 in Fig. 1. (*a*) Balance the resistances for zero deflection when steady current is used. (*b*) Connect the coils so that their total inductance is $L_1 + L_2 + 2M$. Reverse the commutator K and measure the ballistic deflection d_1. (*c*) Connect the coils so that their total inductance is $L_1 + L_2 - 2M$. Reverse the commutator K and measure the ballistic deflection d_2. (*d*) Rearrange the connections according to the scheme indicated in Fig. 2, one coil in each branch of the galvanometer circuit. R_2 is omitted and the total resistance R, from A to B, is made the same as in parts (*a*), (*b*) and (*c*). After balancing the resistances for zero deflection with steady current, reverse the commutator K and measure the ballistic deflection d_3. The required ratios are

$$\frac{M}{L_1} = \frac{d_1 - d_2}{d_1 + d_2 + 2d_3}, \qquad \frac{M}{L_2} = \frac{d_1 - d_2}{d_1 + d_2 - 2d_3}. \tag{18}$$

L_1 is assumed greater than L_2 in the derivation of these equations.

Method 16, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of a Third Coil, Null Method. — Coils 1 and 2, whose self-inductances are L_1 and L_2 and whose mutual inductance is M_{12}, are connected according to the scheme indicated in Fig. 6, coil 1 in one branch of the galvanometer circuit. Coil 3, whose self-inductance is L_3, is included in the other branch of the galvanometer circuit. Non-inductive resistance is connected in series with coil 2 and with either coil 1 or 3 as may be necessary. (*a*) Balance the resistances in the branches of the galvanometer circuit for zero deflection, using steady current. (*b*) Adjust the resistance in series with coil 2 to obtain zero ballistic deflection when the commutator K is reversed. If L_1 is greater than L_3 the magnetic fields of coils 1 and 2 must be opposed; if L_1 is less than L_3 the fields must coincide in direction. (*c*) Transfer coil 3 to a position in series with coil 1, leaving coil 2 in its former place, and again balance the resistances for steady currents. (*d*) Repeat adjustment (*b*) with the new arrangement of coils. The magnetic fields of coils 1 and 2 are now opposed in direction. If R_1 and R_2 are the total resistances from A to B by way of the branch circuits containing coils 1 and 2 respectively, for adjustment (*b*); and R_1' and R_2' are the corresponding resistances for adjustment (*d*); the required ratio of inductances is

$$\frac{L_3}{M_{12}} = \frac{R_1' R_2 \mp R_1 R_2'}{2 R_2 R_2'}. \qquad (19)$$

The choice of sign depends upon the relative magnitudes of L_1 and L_3.

Method 17, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of a Third Coil.—The mutual inductance of two coils, M_{12}, is to be compared with the self-inductance of a third coil L_3. Connect coils 1 and 2 in series in one branch of the galvanometer circuit; and coil 3 in the other branch. By preliminary trial test the relative magnitudes of L_3 and the total inductance $L_1 + L_2 + 2M_{12}$. Connect non-inductive resistance in parallel with the larger of these inductances, and determine according to method 3 the ratio $(L_1 + L_2 + 2M_{12})/L_3$. This assumes that coils 1 and 2 are connected so that their magnetic fields coincide in direction. Keeping coils 1 and 2 in series, interchange the terminals

of one of them so that their fields are opposed, and determine by the same method as before the ratio $(L_1 + L_2 - 2M_{12})/L_3$. The difference between the ratios thus determined is four times the required ratio of M_{12} to L_3.

Method 18, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of a Third Coil, Deflection Method.—Connect the three coils as in the last method, omitting the resistance in parallel with the larger total inductance. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Connect coils 1 and 2 so that their magnetic fields coincide in direction and measure the ballistic deflection d_1, obtained when the commutator K is reversed. (*c*) Connect coils 1 and 2 so that their magnetic fields are opposed in direction and measure the ballistic deflection d_2, obtained when the commutator is reversed. (*d*) Transfer coil 3 to a position in series with coils 1 and 2, altering resistances in series so that the total resistances from A to B by either branch of the circuit may be the same as they were in measurements (*b*) and (*c*). With the magnetic fields of coils 1 and 2 coinciding in direction, measure the ballistic deflection d_3, obtained when the commutator is reversed. (*e*) With the coils arranged as in (*d*), except that the fields of coils 1 and 2 are now opposed in direction, measure the ballistic deflection d_4. The required ratio is

$$\frac{M_{12}}{L_3} = \frac{(d_2 - d_1 - d_3 + d_4)}{2(d_1 + d_2 - d_3 - d_4)}. \qquad (20)$$

The signs in this equation are arranged in accordance with the supposition that L_3 is greater than $L_1 + L_2 + 2M_{12}$. Other relative magnitudes will require other signs.

Method 19, Comparison of Self-Inductance with Capacity, Null Method. — The coil whose self-inductance is L is connected in one branch of the galvanometer circuit, according to the scheme indicated in Fig. 1. Non-inductive resistance is included in series with the coil. The condenser whose capacity is C is connected in parallel with any required portion of this resistance, by means of traveling plugs. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Shift the traveling plugs to obtain zero ballistic deflection when the commutator K is reversed. If R is

the resistance then bridging the terminals of the condenser, the ratio of self-inductance to capacity is

$$\frac{L}{C} = R^2. \tag{21}$$

Method 20, Comparison of Self-Inductance with Capacity, Niven's Null Method.[1] — Branch circuits are made up between points A and B, including on one side a non-inductive resistance, one coil of the galvanometer and the inductance L, in order; on the other side a non-inductive resistance, the other coil of the galvanometer and a second non-inductive resistance. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Connect the condenser whose capacity is C, one terminal to a traveling plug which is placed at a point in the non-inductive resistance, in the branch in which L is found, near the galvanometer; the other terminal to a traveling plug which is joined to the opposing resistance at a point further from the galvanometer. Adjust the traveling plugs to obtain zero ballistic deflection of the needle of the galvanometer when the commutator K is reversed. If R_1 is the resistance from point A to the first plug, and R_2 is the resistance from A to the second plug, the required ratio is

$$\frac{L}{C} = R_1^2 - R_2^2. \tag{22}$$

Method 21, Comparison of Self-Inductance with Capacity, Deflection Method. — Connect the coil whose self-inductance is L according to the scheme indicated in Fig. 7. Capacity C is to be joined in parallel with either one of two equal resistances 1 and 2, each R ohms. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Measure the ballistic deflection d_1, resulting from reversal of the commutator K when the condenser is in the position 1. (*c*) Measure the ballistic deflection d_2, when the condenser is in

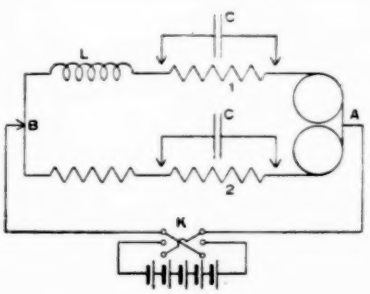


Fig. 7.

[1] Phil. Mag. (5), Vol. 24, p. 231, 1887.

the position 2. The required ratio of self-inductance to capacity is

$$\frac{L}{C} = R^2 \cdot \frac{d_2 + d_1}{d_2 - d_1}. \tag{23}$$

Method 22, Comparison of Mutual Inductance with Capacity, Null Method. — The two coils whose self-inductances are L_1 and L_2, and whose mutual inductance is M, are connected in series, their magnetic fields either coinciding in direction or opposed. The total inductance of the coils, $L_1 + L_2 \mp 2M$, is then compared with the capacity C by method 19. If R_1 is the resistance in parallel with the condenser, required for zero ballistic deflection when the inductance is $L_1 + L_2 + 2M$; and R_2 is the resistance when the inductance is $L_1 + L_2 - 2M$; the required ratio is

$$\frac{M}{C} = \frac{R_1^2 - R_2^2}{4}. \tag{24}$$

Method 23, Comparison of Mutual Inductance with Capacity, Deflection Method. — The coils whose mutual inductance is M are connected in series and placed in the position occupied by the inductance L in Fig. 7. The capacity C is connected in parallel with either of the resistances 1 or 2, which need not be equal. (*a*) Balance the resistances for zero deflection of the needle of the galvanometer, using steady current. (*b*) Measure the ballistic deflection d_1, resulting from reversal of the commutator K, when the coils are connected for total inductance $L_1 + L_2 + 2M$, the condenser being disconnected. (*c*) Measure the ballistic deflection d_2, resulting from reversal of the commutator when the total inductance is $L_1 + L_2 - 2M$, the condenser still disconnected. (*d*) Measure the ballistic deflection d_3, when the condenser is connected in parallel with resistance 1 and the total inductance is $L_1 + L_2 + 2M$. (*e*) Measure the ballistic deflection d_4, when the condenser is in parallel with resistance 2 and the total inductance is $L_1 + L_2 + 2M$. If R_1 and R_2 are the magnitudes of the resistances 1 and 2, the required ratio of mutual inductance to capacity is

$$\frac{M}{C} = \frac{R_1^2 + R_2^2}{4} \cdot \frac{d_1 - d_2}{d_4 - d_3}. \tag{25}$$

Other Applications of the Differential Ballistic Galvanometer.

In addition to the methods of measurement described in this paper, many of the published methods of determining inductance and capacity which make use of the Wheatstone network, may be carried out with the aid of the differential ballistic galvanometer. The Rayleigh-Maxwell method of measuring self-inductance may be taken as an example. Four resistances, three non-inductive and one containing the inductance to be determined, form a quadrilateral, whose diagonals are occupied by a battery and a galvanometer respectively. If the galvanometer is replaced by a short wire, and the coils of a differential galvanometer are included with two of the resistances of the quadrilateral, one each side of a junction with the short wire, the measurement of inductance may be made as usual, in terms of resistance, deflection and period of vibration of the needle of the galvanometer. When the resistance of the galvanometer is small, there is no apparent gain in the use of the differential arrangement. When its resistance is large, then for given resistances and battery power, nearly twice the ballistic deflection may be obtained with the differential galvanometer, that would result from the use of the same galvanometer, its coils connected in series and employed as a ballistic galvanometer of the ordinary type. When the inductance to be measured is small and the resistance associated with it is large, relatively large current must be used to secure ballistic deflections which can be measured. Under these circumstances there is difficulty in maintaining balance of resistances when steady current is flowing, because of unequal heating effects in different parts of the circuit. Lessening the current required to produce a given deflection, decreases the heat liberated in the circuit in proportion to the square of the current, and tends to obviate the difficulty described.

Comparing the Rayleigh-Maxwell method with method 10 of the list now offered, the latter method has the advantage of greater sensitiveness. This may be established by comparison of the expression for the discharge through the galvanometer in the former method, with equation (4) of this paper. It is assumed for this comparison that the same amount of wire is wound in a single coil,

or differentially in two coils, and that the steady current flowing through the inductive resistance is the same in both methods. The advantage of greater sensitiveness, shared by other methods here described, increases in general with higher resistance of the galvanometer coils. The writer is convinced that partly because of greater simplicity of connections, leading to greater ease of manipulation, partly because of increased sensitiveness or smaller steady currents required for the same deflection of the galvanometer needle, the methods now presented will show some real gain in precision, over the methods which make use of the Wheatstone quadrilateral and a ballistic galvanometer of the ordinary type.

UNIVERSITY OF CALIFORNIA,
March, 1905.

MAGNETIC EFFECT OF ELECTRIC DISPLACEMENT.

By J. B. Whitehead.

Part I.

In two former papers[1] the writer described a series of experiments which were undertaken as a test of the validity of Maxwell's assumption of the presence of a magnetic effect accompanying an electric displacement in a dielectric. The results of the experiments have been shown by Kolacek[2] to be in accord with Maxwell's theory. From the symmetry of the Maxwell equations we must expect an electrical effect or force accompanying magnetic displacement (if we may use the analogous expression). Thus in the writer's experiments the resulting mechanical force acting on the dielectric was shown by Kolacek to be

$$\frac{K}{4\pi V}\frac{d}{dt}ZM;$$

where K is the dielectric constant, V the ratio of the two systems of units, Z the electric intensity in electrostatic and M the magnetic intensity in electromagnetic units. Since in the experiments both Z and M were periodic functions differing in phase by 90° the mean value of over a period the force was zero, and therefore there was no motion of the dielectric. This result then is a proof of the existence of the electric effect of the "magnetic displacement current," if the magnetic effect of electric displacement be granted. Evidence of the existence of the latter has recently been given by the work of Eichenwald,[3] although many other experimenters have sought it with negative results. It is therefore desirable that further evidence be had, and the present work was undertaken with the aim of observing the two effects independently.

[1] Mag. Eff. of Elec. Disp. Report to Carnegie Institution, 1904, and Phys. Zeits., 4, 229, 1903; 5, 300, 1904.

[2] Phys. Zeits., 5, 45, 1904.

[3] Annalen der Physik, 6, 1903.

The following experiment was devised for showing directly the magnetic effect of electric displacement.

If a portion of dielectric be subjected to an alternating electric intensity, alternating displacement currents will be set up in the dielectric. If, now the dielectric be properly surrounded by a magnetic circuit, alternating magnetic induction will be set up therein, and if this circuit be wound with a number of turns of wire, an alternating E.M.F. will be induced therein, and this E.M.F. may set up a current. The value of the current may be calculated and measured. As constructed the dielectric was a cylinder of paraffin. The E.M.F. was applied to electrodes consisting of circular brass plates on the end faces of the cylinder. Concentric with the cyinder and surrounding it was the laminated magnetic circuit consisting of soft iron disks; the iron ring thus formed was wound with what may be called the secondary winding, in circuit with which the measuring instrument was connected.

Preliminary calculations indicated that the apparatus would have to be of large size, and also that means for measuring very small values of alternating current must be had, if the expected effect was to be detected.

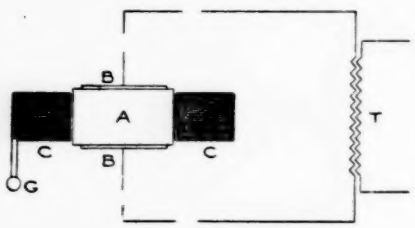


Fig. 1.

Consider the apparatus as shown in a diametral section in Fig. 1. A is the cylinder of dielectric, BB are the electrodes, CC the surrounding magnetic circuit. If E is the difference of potential between the electrodes, E/l is the electric intensity active on the dielectric, l being the length of the cylinder. The density of the displacement current is then

$$q = \frac{K}{4\pi l}\frac{dE}{dt}.$$

If $E = E_m \sin \alpha$, $q = \frac{K}{4\pi l} E_m \frac{d\alpha}{dt} \cos \alpha$; and since $\frac{d\alpha}{dt} = 2\pi N$

$$q = \frac{K}{4\pi l} E_m 2\pi N \cos \alpha.$$

If a be the area of the circular section of the cylinder, the total current is

$$i = \frac{KE_m Na}{2l} \cos a,$$

if E is expressed in volts and i in amperes

$$i = \frac{KE_m Na 10^8}{2l(3 \times 10^{10})^2 \times 10^{-1}} \cos a.$$

In the experiments $K = 2$, $E_m = \sqrt{2} \cdot 25{,}000$, $N = 133$, and $a = \pi \cdot 19.25^2 = 1{,}164$ sq. cm.; $l = 20$ cm. The maximum value of the current is therefore

$$i = \frac{2 \times 25{,}000 \times 133 \times 1{,}164 \times 10^8 \times \sqrt{2}}{2 \times 9 \times 10^{19} \times 20} = 3.03 \times 10^{-4} \text{ amperes},$$

$$i \text{ (\textit{effective})} = 2.15 \times 10^{-4}.$$

The magnetic effect of this current in a surrounding magnetic circuit will depend on the distribution of the current in the dielectric. The most obvious assumption is that the effect is the same as though the current were all concentrated at the center, *i. e.*, on the axis of the cylinder. We may then calculate the magnetic intensity at any distance from this axis, assuming that the wires leading to the electrodes, and the dielectric are equivalent to a long straight conductor lying along the axis of the apparatus. The leading-in wires were accordingly long and straight. We have then:

Maximum value of H, the magnetic force at a radius of 20 cm. $= 2i/10 = 3.03 \times 10^{-6}$, at radius of 40 cm. $H = 1.51 \times 10^{-6}$. The magnetic circuit was composed of rings of annealed sheet armature iron, 41.4 cm. internal and 82.8 cm. external diameter. Built up and compressed the depth was 15.5 cm.; the cross section was thus 320 sq. cm. The weight was 900 lbs. The average value of H may then be taken as 2.26×10^{-6}. The wires leading to the electrodes continued straight away for 100 cm. on each side. They then turned through a right angle and were led parallel to each other to the transformer 5 meters away. The correction factor for this departure from the infinite straight conductor may readily be shown

to be only $.09i/r$, for the magnetic force as calculated from the expression $H = 2i/r$; thus the average value is $H = 2.09i/r$. Using the values for i and r given above, the average value of H becomes 2.36×10^{-6}.

$$\therefore \text{ Total Magnetic Induction} = 2.36 \times 10^{-6} \times \mu \times 320.$$

Data as to the permeability of soft iron in laminations at such low values of the magnetic intensity are meagre.

Values of μ for small magnetizing forces have been determined by Baur[1] and Lord Rayleigh.[2] The latter carried the field from 4×10^{-2} down to 4×10^{-5}, and showed that μ was constant throughout this region; there is nothing to indicate that it will change for values of H still nearer to O. This value for soft iron is 183, as obtained with constant magnetic force on bars. Ewing states that for laminated structures there is little if any viscosity, or lag, at small forces.

If we assume this value we have:

$$\text{Total Induction} = 2.36 \times 10^{-6} \times 320 \times 183 = 1.35 \times 10^{-1}.$$

A secondary winding consisting of 966 turns of No. 18 B. & S. wire completely surrounded the magnetic circuit. A similar winding of the same number of turns was afterward wound over this, the two coils beginning and ending at the same point. The E.M.F. induced in one of these windings is:

$$E = 1.35 \times 10^{-1} \times \pi\sqrt{2} \times 133 \times 10^{-8} \times 966 \text{ volts},$$
$$= 7.8 \times 10^{-4} \text{ volts}.$$

If we can measure a current due to this E.M.F. we have a test of the validity of the theory subject to the assumptions already stated. The Rubens Vibration galvanometer[3] was used as measuring instrument. Its principle is the adjustment of the tension of a stretched wire so that its natural frequency of oscillation is the same as that of the electric circuit. The wire carries a row of small magnets in suitable relation to poles carrying coils traversed by the cur-

[1] Wied. Ann. XI., 1880, p. 399.
[2] Phil. Mag., March, 1887.
[3] Wied. Ann., No. 56, p. 27.

rent to be measured. The amplitude of the oscillation, observed by light reflected from a small mirror on the wire is a measure of the current. The instrument was obtained from W. Oehmke, Berlin, and was made from the specifications contained in the article already referred to. It was calibrated by the following method. A low value of alternating E.M.F. of the frequency to be used was impressed on a noninductive resistance of 2,000 ohms; connections were made at two points differing by 100 ohms, and between them the galvanometer, in series with a noninductive resistance of 10,000 to 100,000 ohms, was inserted. The galvanometer being tuned to synchronism for the frequency used, the widths to which a slit of light reflected from the mirror is spread out, corresponding to the various values of current, were read. The filament of an incandescent lamp was used as source, and the spreading of the image was measured on a scale; in the eyepiece of the telescope the scale could be read to tenths. The calibration curve, given by the accompanying table, covers the range of the experiments. The resistance of the instrument was 215 ohms.

Amperes.	Deflections.	Amperes.	Deflections.
4×10^{-5}	4.2	3.3×10^{-6}	.6
2×10^{-5}	2.1	2.8×10^{-6}	.55
1×10^{-5}	1.2	2.5×10^{-6}	.5
6.6×10^{-6}	.95	2.2×10^{-6}	.45
5×10^{-6}	.8	2×10^{-6}	.4
4×10^{-6}	.7		

If now the instrument is connected with the secondary winding ($r = 13$ ohms) by the calculation above, the order of magnitude of the resulting current in it should be $(7.8 \times 10^{-4})/228 = 3.42 \times 10^{-6}$ amps.

The Experiments. — The source of current was an 8-pole single phase alternator giving approximately a sine curve of E.M.F. at 110 volts effective value. It was directly connected to a direct current motor, supplied from 110-volt mains. A heavy flywheel on the shaft and rheostats in the field and armature of the motor, controlled from the point of observation, resulted in a suitable constancy and control of the frequency. The alternator supplied the

primary of a step up transformer of ratio 100/25,000, the high tension terminals being connected to the electrodes. The transformer was at 5 meters distance from the apparatus proper, the wires being parallel and 2.5 meters apart until they intersected the axis of the apparatus. The lower electrode supported the paraffin, and the upper rested on it. The galvanometer was near the magnetic circuit and suitable tests indicated that it was free from electromagnetic disturbances due to alternator, transformer, etc. A reversing switch was placed between the secondary winding and the galvanometer, also a single pole switch which opened one side of the circuit only. A small solenoid placed near the galvanometer, and supplied from the machine, served as a test for synchronism and constancy of conditions. The apparatus as described permits several variations of conditions, giving different values of the current to be expected in the galvanometer. Paraffin or air may be used as dielectric; the secondary coils may be used single, in series, or in parallel. Different values of voltage and frequency were not used, as spurious effects would also probably vary with these quantities.

A typical set of readings is given in the table.

Secondary Coils.	Paraffin.					Air.				
					Mean.					Mean.
Coil No. 1, No. 2 open.	.8	.8	.9	.8	.82	.6	.6	.6	.5	.57
Coil No. 1, No. 2 closed.	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
Coil No. 2, No. 1 open.	.8	.9	.9	.8	.85	.6	.6	.6	.6	.6
Coil No. 2, No. 1 closed.	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
Coils No. 1 and No. 2 in series.	1.3	1.3	1.3	1.4	1.32	.9	.9	.9	.8	.87
Coils No. 1 and No. 2 in parallel.	.7	.75	.8	.8	.76	.5	.5	.5	.6	.52

These readings were unchanged on reversal of galvanometer connections. Primary voltage 113. Frequency 130. The method of observing was as follows: By means of the rheostat in the motor field the speed was gradually brought to the desired frequency, this being indicated by the broadening of the image of the filament on the telescope scale. The machine was then allowed to run free, and the condition of perfect synchronism would hold for several seconds, often longer; it would then drop out for a few seconds and then

return. The readings given represent, in each instance, the observations of at least three of these recurring periods of synchronism. Observations made on different days, the paraffin being removed for the air reading after each set, gave readings in good agreement with those in the table.

It will be noted that the values with air as dielectric are uniformly lower than those for paraffin, as is to be expected. They do not, however, differ in the ratio of the dielectric constants. In view of the uncertainty of the values of these constants at the frequency used, and of the uncertainty of the distribution of the field in the two cases, the discrepancy does not appear serious. More important, however, is the fact that the series arrangement of the two secondary coils does not result in a doubled value of the reading for either coil alone, although it is noticeably greater; a possible explanation lies in the arrangement of the windings of the two coils. As already stated, the two coils were wound in parallel in a single layer. Thus when connected in series half of the total induced E.M.F. existed between each pair of wires. This difference in potential would tend to upset the symmetrical and uniform oscillation of the electrostatic charges induced on the secondary winding, thus resulting in the equivalent of a current which would cause a second induced E.M.F. in the secondary winding. This would not occur in the parallel arrangement, and the readings conform to this view.

That these electrostatic charges introduce no error in the readings is indicated by the observations given in the tables for "closed" coils. The short circuiting of one coil always nullified the E.M.F. in the other. This shows that the current in the galvanometer circuit is electromagnetically induced and is not electrostatic in its origin. Aside from this the symmetrical form and arrangement of the apparatus gives no cause for an unsymmetrical oscillation of the induced charges.

Conclusion. — The calculated effect for one coil reduced to the conditions of the test, *i. e.*, 113 volts and 130 cycles, is 3.16×10^{-6} amps. The current corresponding to the mean values (.82) of the deflections for coil No. 1, as given by the calibration curve, is about 5.2×10^{-6}. The observed effect is thus of the same order of mag-

nitude as that calculated; the uncertainty of the assumptions which have been made may well account for the difference in the numerical values.

PART II.

The following modification of the writer's earlier experiment[1] was suggested by Kolacek for showing the electric effect of magnetic displacement. Place the dielectric in a constant electric field and a magnetic field at right angles thereto; if the magnetic field be suddenly reversed we should according to Maxwell's theory expect an implusive force to act on the dielectric. For the expression for the force now becomes $K/4\pi\, V Z\, \partial/\partial t\, M$, since Z is a constant, *i. e.*, there is no electric displacement current. According to Kolacek this experiment also affords a test of the modifications of Maxwell's theory suggested by Lorentz. The former in expressing the value of the displacement current as $K/4\pi\, \partial Z/\partial t$ says nothing as to the mechanism in the dielectric; simply that the impression or relief of electric tension or polarization in a dielectric of specific inductive capacity K is equivalent in magnetic effect to current of density $K/4\pi\, (\partial Z/\partial t)$. Lorentz, however, assumes that the displacement current is made up of two parts, first the polarization of the ether, and second the displacements of atomic or ionic charges within the molecules of the dielectric; that these being actual motions of charges are equivalent to currents and that Maxwell's expression $K/4\pi\, (\partial Z/\partial t)$ is equivalent to $1/4\pi\, (K_e + \Sigma c)\, \partial Z/\partial t$, where K_e is the value of K for the ether and Σc is the summation of the molecular convection currents. This assumption admits the magnetic effect of electric displacement, but in the proposed experiment in which we should expect a deflection according to Maxwell, the electric field is constant, the convection currents are absent, and therefore according to Lorentz there would only be reaction on the ether which would cause no deflection of the matter of the dielectric.[2] This conclusion has been questioned by R. Gans[3] who concludes that according to Lorentz a deflection is to be expected, but that it would have a value $K - 1/K$ times that based on Maxwell's original theory.

[1] Phys. Zeits., 4, 229, 1903.
[2] Kolacek, Phys. Zeits., 5, 455, 1904.
[3] Phys. Zeits., 5, 162, 1904; 5, 627, 1904.

A calculation of the magnitude of the impulsive force for a specific case indicated (as is shown below that) it would be so small that its effect would be difficult if not impossible to observe directly. Nevertheless it was decided to carry the experiment through with as favorable conditions as possible, in order to see how nearly a calculated effect might be approached by increasing as far as possible several factors.

Apparatus. — As dielectric a cube of rock salt 1 cm. on an edge was used. This was first suspended between two vertical parallel plates at different potentials; it was found that in this arrangement the dielectric had no equilibrium position owing to the irregular distribution of the field; the following arrangement was therefore adopted. The dielectric was attached to the center of a light glass rod, which was suspended by two silk fibers so as to lie along the axis of a circular coil. Two horizontal parallel brass plates were placed respectively above and below the dielectric, and being suitably supported and insulated constituted the electrodes. The dielectric was thus suspended in a magnetic field set up by the coil, and in an electric field set up by the electrodes at right angles to the magnetic field. The experiment then consists in reversing the magnetic field, the electric field being kept constant, and looking for a ballistic deflection of the block of dielectric. The force on the dielectric is as already stated, $K/4\pi Z\partial/\partial t M$, K being the dielectric constant, Z, the electric and M the magnetic intensity. These quantities were made as large as possible. Rock salt was chosen as the dielectric, the value for K being taken as 5. The coil for setting up the magnetic field consisted of 1,200 turns of No. 18 B. & S. wire; its internal diameter and its length were each 7.6 cm. The magnetic intensity at the center as measured by an exploring coil and alternating current was found to be 166 per ampere. In the experiments the current, limited by heating, was carried to 6.5 amperes; hence $M = 1{,}080$. Several plans for increasing the electric intensity in the dielectric were tried. The upper and lower faces of the dielectric were covered with metallic foil, a small strip extending from each parallel to the glass supporting arm. The electrodes described above being removed, very light hangers of fine wire were arranged to hang with small pressure against the

strips projecting from the metal coatings of the dielectric faces, and so connect these coatings with the terminals of an induction machine. The slight pressure due to the hangers was in the same direction as the expected deflection. On charging, the little hangers were statically repelled, and if made heavier, the same effect set the dielectric in motion. After several modifications this method was abandoned as impracticable. Light wires from the metal coatings, dipping into cups of liquid conductor were then tried. In this case inequalities of surface tension prevented stationary conditions. The plan of suspending between horizontal electrodes was then adopted, the electrodes being brought as near together and the potential difference carried as high as possible, *i. e.*, until a greater value resulted in a discharge. The dielectric cube was 1 cm. on an edge, and the minimum distance between electrodes consistent with steady conditions was 2.2 cm. The potential difference was carried to 15,500 volts, as measured by the sparking distance between brass spheres, using the tables of Paschen [1] for calculation. The sensibility of the apparatus depends principally on the length of the suspending threads. They consisted of very fine silk fibers and their length was 112 cm. The whole apparatus was suitably enclosed, the suspending fibers occupying separate glass tubes.

We have then as values of the various quantities: Distance between electrodes 2.2 cm. Maximum potential difference at electrodes $E = 15{,}500$ volts. Maximum intensity of magnetic field $M = 1{,}080$. Length of suspension 112 cm. Dielectric cube 1 cm. on an edge. Mass of suspended system about 3 grams. We have then:

$$\text{Electric intensity in dielectric } Z = \frac{E}{ld + Kla} = \frac{15{,}500}{1 + 1.2K}.$$

If the magnetic field is reversed the consequent impulse on the dielectric is:

$$\int A\,dt = \frac{K}{4\pi V} Z \int dM$$

$$= \frac{5}{12.5 \times s \times 10^{10}} \cdot \frac{15{,}500 \times 10^{8}}{7} \cdot \frac{2{,}160}{3 \times 10^{10}}$$

$$= 2.1 \times 10^{-7} \text{ c.g.s.}$$

[1] Wied. Ann., XXXVII., p. 79, 1889.

If we divide the moment of this impulse by the moment of inertia of the suspended system we have the consequent angular velocity of the dielectric cube. From this we derive its kinetic energy and hence the arc through which it will swing against the force of gravity. A calculation of this quantity gives for the deflection a magnitude of the order 10^{-6} cm., which not only explains the uniformly negative results of the experiments, but indicates that in this form the experiment suggested by Kolacek cannot be expected to give evidence on the questions at issue.

The experiments were performed under the provisions of a grant from the Carnegie Institution.

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THE INFRA-RED SPECTRUM OF CO_2 AND NITROGEN.

BY E. R. DREW.

THE present investigation consists chiefly of an attempt to determine the source of the band which the writer[1] found at 4.75 μ in the radiation from the positive column of a Geissler-tube discharge in air. I had thought it probable that the band was due to nitrogen, but Professor Warburg suggested that it might be due to CO_2, as it is in the neighborhood of the well-known band at 4.4 μ, and no special precautions had been taken to get rid of this persistent gas. Further, Paschen[2] found that the position of this band varies with the temperature, from 4.27 μ at 17° to 4.4 μ in the Bunsen flame. It was, therefore, of interest to see whether it would be still farther displaced, toward the longer waves, in the radiation from the Geissler tube, in which the relatively high optical efficiency points to emission under conditions which would correspond to exceedingly high temperatures in flames. The same reasoning would require the wave-length to become greater as the pressure of the gas in the tube decreases.

The results obtained are, briefly, that a tube filled with CO_2 shows a maximum at 4.67 μ when the pressure is 3 mm., and at 4.70 μ at 0.6 mm. — these being the extreme pressures which could be used. The much smaller band which occurs at 2.7 μ in the absorption spectrum of CO_2 at 17° also gave deflections which were unmistakable, but not large enough to locate it with certainty. Its approximate wave-length is 2.47 μ. The tube was then filled with air, and later with nitrogen, freed from CO_2, with which there was no evidence of CO_2 in the color of the discharge, or in the visible spectrum. There was then no trace of the small CO_2 band, but there was again a maximum at about 4.7 μ, which, however, showed no evidence of displacement with change of pressure.

[1] PHYS. REV., XVII., p. 321, 1903.

[2] Wied. Ann., 53, p. 334, 1894.

Apparatus. — A mirror spectrometer with rock-salt prism was used, with the mirror-prism device of Wadsworth, it being convenient to keep the telescope and collimator tubes fixed. This allowed an approximately air-tight house to be built about the prism, in which drying material was kept. The prism surfaces thus remained in good condition throughout.

The vacuum-tube is shown in section in Fig. 1. The collimator slit, at *S*, was 0.35 mm. wide and 6 mm. long, cut in a piece of sheet aluminium, and placed 2 mm. behind a somewhat larger opening in the anode, also of aluminium. The constricted portion of the tube in front of the anode is of elliptical section, 3 x 8 mm., enlarging toward the cathode, which is an aluminium cylinder, entirely shielded from the slit by the shoulder of the tube. It was hoped by this means to secure the greatest possible intensity in the radiation passing through the slit. A serious disadvantage proved to be the fact that this intensity depended chiefly upon the distribution of current just in front of the anode, and this was liable to change at any time. A series of readings was often thus spoiled when half finished; and series of readings which were in themselves consistent, could seldom be compared with each other. The tube was set in the collimator tube so that the slit came in the proper position, the inner end closed by a fluorite plate and the outer by a rather rough plate of rock-salt, through which radiation from an outside source could be passed through the slit.

Fig. 1.

It was impossible, on account of magnetic disturbances, to use a thermopile as receiving instrument, so a radiometer was constructed. In an aluminium plate placed just inside the fluorite window, a slit exactly like the collimator slit allowed the radiation to fall upon one of the vanes. The whole inner face of this plate, including the slit, was covered by a thin plate of mica. The maximum elongation was reached in about 30 seconds, and the instrument was about twice as sensitive as the one used by the writer at Ithaca.

The chief difficulty in the investigation arose from the fact that it was necessary to use a form of Julius suspension for the radiometer,

to free it from the tremors of the building. It was thus not a fixed part of the spectrometer, and its relative position varied slightly from time to time.

Current was obtained from a storage battery of 1,250 volts.

Observations. — The difficulty described above made necessary the adoption of a rather laborious method for taking observations. This consisted in simultaneously locating the maximum for the radiation from the discharge in the tube, and for that from the Bunsen flame, by reading a deflection first for the one and then for the other at nearly corresponding points on the two curves. The radiation from the flame passed through an aperture 4 mm. in diameter in a screen 30 cm. from the slit, reaching the latter through the rock-salt window on the outer end of the tube. The radiometer deflections were then of the same order of magnitude for the two sources.

The refractive indices given by Martens[1] were used to construct a calibration curve for the prism, of refracting angle 60° 9′. The measured angle between the *D*-line and the 4.4 μ maximum agreed with the curve to within the errors of the settings. The smallest vernier reading was 0.5′, corresponding to 0.066 μ in this part of the spectrum, so it was desirable to locate the maxima to within 0.1′ if possible.

When the two slits are equal and parallel, so that the image of one with monochromatic light falls exactly upon the other, the plotted deflections caused by a line or very narrow band will give an isosceles triangle. A series of careful tests with the Bunsen flame showed that these conditions were nearly enough fulfilled to justify the location of a maximum by the intersection of two straight lines drawn through the points representing three or four deflections on each side of the maximum. It was usually obvious that from one to three of the largest deflections did not belong to the triangle, so these were left out of account in the plotting. In one trial of the method, three sets of readings were made for the flame, two simultaneously, the third immediately after, and the extreme difference among the results for the position of the maximum was 0.07′.

[1] Drude's Ann., 6, 623, 1901.

It is hardly necessary to reproduce all of the plots on which the results are based. Their character is well enough indicated by the samples given in Fig. 2. In each case the upper plot is for the flame, while the lower is for the gas used, as follows:

A. CO_2, pressure 0.6 mm. *C*. Air, pressure 0.6 mm.
B. " " 3.0 " *D*. " " 3.0 "

A represents one of the earlier series of readings taken, and is one of the poorest. In each plot the abscissas are circle readings in minutes of arc, the ordinates deflections in millimeters on a scale at 100 cm. The current in all cases was 0.01 ampere.

With CO_2 the pressure increases rapidly on first starting the cur-

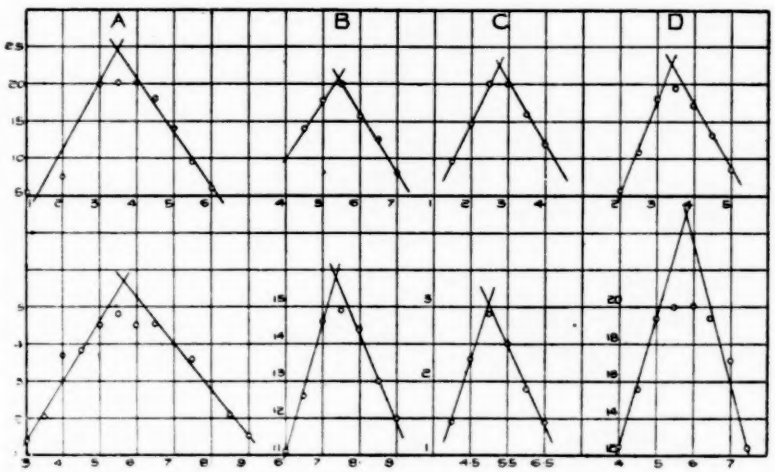

Fig. 2.

rent, on account of the decomposition of the gas into CO and O, which according to Collier[1] may amount to 65 per cent. After a few minutes a fairly steady condition is reached, and there is no further trouble.

All of the gases used were dried by passing through strong H_2SO_4 and a long tube of P_2O_5. The air and nitrogen were freed from CO_2 by passing through a KOH tube. The gas was then pumped through the tube, while a current more than twice as strong as that used in the measurements was passed through it continuously

[1] Jour. Chem. Soc., 465, 1063, 1901.

for more than an hour, keeping the tube quite hot, until the discharge showed none of the characteristic color due to the carbon bands, and none of these were visible in the spectroscope. It was then thought hardly probable that the weaker current, which warmed the tube only slightly during the 30 seconds required for a deflection, would set free enough CO_2 from the tube walls or the electrodes to give any very strong radiation. The only prominent lines seen in the spectroscope were the red hydrogen, and a green line, probably that of mercury.

Nitrogen was prepared from sodium nitrite and aumonium sulphate, left standing over water for 24 hours, and passed through pyrogallic acid solution to remove traces of oxygen.

In Fig. 3 the lower plot shows the small CO_2 band, at 3 mm.

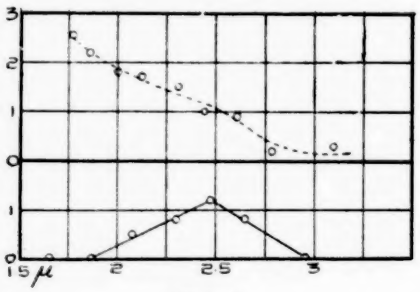

Fig. 3.

pressure, its intensity being roughly 0.07 that of the large maximum. The upper dotted curve shows the deflections obtained in the same region with nitrogen, which shows no trace of the CO_2 band.

The numerical results given below express the distance of each maximum from that of the Bunsen flame in minutes of arc. The factor given after each result is a weight assigned to it in averaging, derived from the appearance of the plot, and the possible variation in the result caused by permissible variation in the plotting. The two starred results, which were obtained with nitrogen, agree so closely with those for air, that there seems no reason for treating them separately.

The mean results are below expressed in wave-lengths, assuming 4.40 μ for the flame maximum.

CO_2				Air.			
0.6 mm.		3.0 mm.		0.6 mm.		3.0 mm.	
2.20	4	2.25	4	2.33	3	2.35	8
2.21	1	1.96	2	2.25	10	2.19	5
2.52	1	1.99	10	2.11	9	*2.18	10
		2.04	8	2.18	4	*2.19	7
Mean 2.25		2.05		2.21		2.23	
4.697 μ		4.67 μ		4.692 μ		4.695 μ	

The radiation on its way through the spectrometer passes through nearly a meter of air, and suffers absorption by the CO_2 contained therein. The maximum for this absorption, according to Paschen, lies below 4.3 μ, so that its effect upon the location of the emission maxima will be small. It will also be very approximately the same in all cases, hence the apparent displacement of the CO_2 band with the pressure in the tube would not be affected.

On account of the previously mentioned fluctuations in intensity of the radiation passing through the slit it was impossible to make any definite comparison of the intensities of the maximum with the different gases used. There was no difference great enough to be certainly detected with the same current and pressure.

It was not thought worth while to try oxygen, since Ångström[1] found that its total radiation was not more than one tenth of that from nitrogen which was absorbed by his alum plate, and hence probably not more than one third of that given by this band alone.

It is possible that the maxima may be due to some impurity common to all the gases used; but it seems highly probable that those found with CO_2 are really due to that gas, while that found with air and nitrogen is not a CO_2 band.

From Paschen's results for the change in wave-length of the CO_2 band, from 4.27 μ at 17° to 4.4 μ in the Bunsen flame, the farther displacement to 4.7 μ in the vacuum tube seems reasonable on considering the character of the radiation, which here has a considerable portion within the visible spectrum, while in the flame it all lies comparatively far in the infra-red. An extrapolation of Paschen's results, which would be rather rash, would lead to the speculation that

[1] Wied. Ann., 48, 509, 1893.

radiation similar to that of the vacuum tube might be gotten from CO_2 if it could be raised to a temperature of the order of 10,000°, provided that its properties were then the same as in the vacuum tube, which is not in the least likely.

I take pleasure in expressing my thanks to Professor Warburg for placing at my disposal the facilities of the Physical Institute, and for many valuable suggestions during the investigation.

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